King Edward VI Camp Hill School for Girls

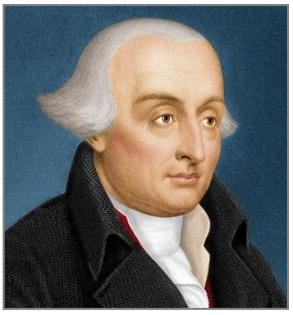
Maths Department Newsletter

12th September 2023

97, 907, 9007, 90007 and 900007 are all happy primes

News

Welcome to the latest edition of the maths newsletter. I hope you had a good summer holiday. One of the things I did over the summer was read a bit about the famous mathematician Joseph Louis Lagrange. If you are new to the school, you won't know this yet but one of the topics we study in year 7 is called Number Theory. This is the topic where we look at how whole numbers are related to each other. We look at prime numbers, factors and multiples, divisibility rules and that sort of thing. One of the things Lagrange is famous for is his 'Four Square Theorem'.[1] This is the fact that every positive whole number can be made by adding together the squares of four other numbers. For example, the numbers 145 and 10 can be made in the following ways:



$$145 = 2^2 + 4^2 + 5^2 + 10^2$$

$$10 = 3^2 + 1^2 + 0^2 + 0^2$$

But (you might be thinking), how do we know you can do that for every whole number? All we have shown is that we can do it for two numbers. How do we know that if we chose a different number at random, we would still be able to do it? This is a good question. The reason we know is that Lagrange proved it. If you would like to know how he proved it, I'll tell you on the back of this newsletter.

Puzzle

Here is a great puzzle that requires a bit of algebra.

If x and y are positive integers[2] and

$$x + xy + y = 54$$

what is the value of $x + y$?

To solve this, you will need to do a bit of factorising, and use some of your knowledge about prime factorisation.

Joke

Visitor: "How old is that Tyrannosaurus Rex skeleton?"

Guide: "70,000,006 years."

Visitor: "Wow. How can you be so precise?"

Guide: "They told me it was 70,000,000 years old when I started working here and I've been here 6 years."

1. He is famous for lots of things but I find his four square theorem particularly interesting. When a group of professional mathematicians were asked 20 years ago to vote for what they thought were the most important mathematical theorems of all time, this one came 19th.
2. Integers are whole numbers.

Lagrange's Four Square Theorem

OK, so I said on the front of this newsletter that I would tell you on this side of the newsletter how Lagrange proved that every positive whole number can be written as the sum of four squares.

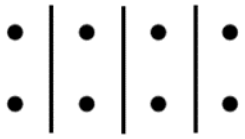
The truth is that I can't really do that, as the argument that he used is too long and too complicated. What I thought might be a fun thing to do, though, is to explain the proof to you over the course of several maths newsletters. In doing this, it will give you a real understanding of how an interesting theorem is proved and, on the way, we can think about lots of other mathematical ideas too. It's an experiment. It might not work but I think it might be good to try, so let's start now and see how we go.

Odds and Evens

My aim is to get through this proof without ever really having to talk about anything difficult. By that, I don't mean that I won't talk about anything deep or interesting; I just mean that I'm going to try to break things right down so that you never feel stuck or confused. Let's start by thinking about odd and even numbers.

Whole numbers can be categorised in lots of different ways. One of those ways is to say that each one of them is either odd or even. Even numbers are those numbers that can be divided exactly into two equal parts which are also whole numbers. For example, 8 is even because it can be divided into two groups of 4.

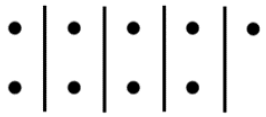
It can also be divided into 4 groups of 2. This is not quite the same thing.



This means we have a second way of thinking about what an even number is. It is a number that can be divided up into groups of 2 without there being anything left over.

In maths there is often more than one way of looking at something, and it is sometimes the case that understanding a problem which seems difficult can suddenly become easy if you just change the way in which you look at it.[3]

All the whole numbers that are not even are called odd. These are numbers that, if you start taking away groups of 2 from them, you will always end up with 1 left over. The number 9, for example, is odd.



We can think of an odd number as an even number with 1 added to it.

GCSE Question

These ideas are directly relevant to how you solve some GCSE questions on proof. See if you can solve this one:

Prove that the sum of the squares of any two consecutive odd numbers is always 2 more than a multiple of 8.

Let us know if you get stuck.

3. This is a really important mathematical principle. It will be very useful when you are doing your GCSE Maths exam.