

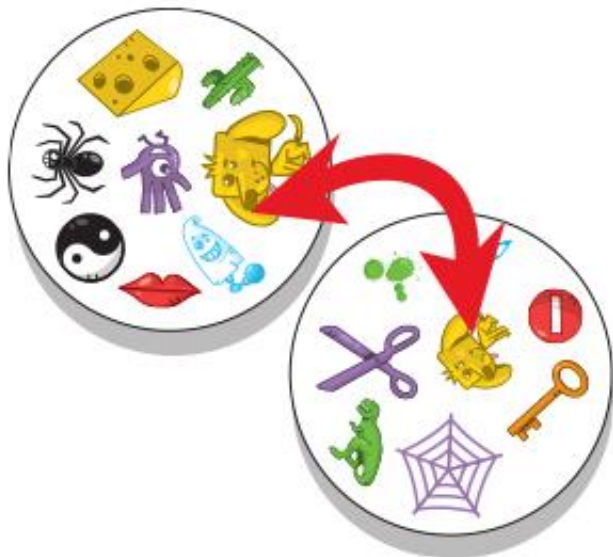


93 is the maximum number of pieces a cake can be cut into using 8 straight cuts

News

Happy New Year, and welcome to the first maths newsletter of 2022.¹ The weather's cold and chances are you will have spent some time recently playing a board game² or a game of cards with your family.

Many of you will probably be familiar with a card game called Dobble. If you have never played it before, each of the cards has eight symbols, and players race to be the first to spot a symbol which appears on both cards³.



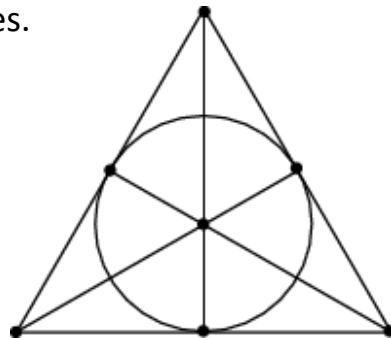
There are 55 cards in the pack and every pair of cards has a symbol in common. But what's really fascinating is that every pair of cards has **exactly** one shared symbol, no more. There's a lot of maths going on behind the scenes!

Maths Words

A **projective plane of order n** is a set of $n^2 + n + 1$ points and $n^2 + n + 1$ lines with the following properties:

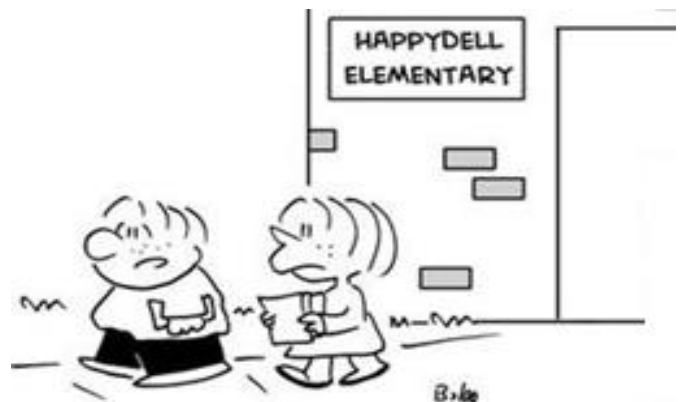
1. Every point is on $n + 1$ lines.
2. Every line contains $n + 1$ points.
3. There is a line through every pair of points.
4. Every pair of lines intersect at exactly one point.

The **Fano plane** (pictured below) is the smallest projective plane, it has order 2. Check that it satisfies each of the four properties.



Read on to find out what this has to do with the game Dobble!

Joke

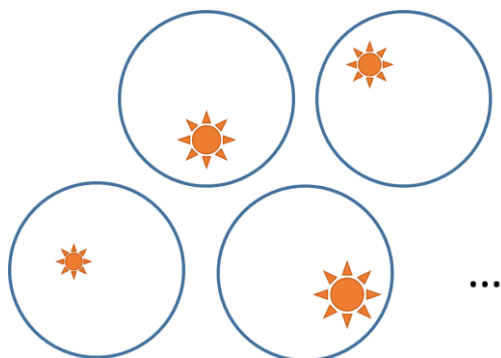


"Plane geometry is hard enough — what will fancy geometry be like?"

1. 2022 is another skinny number. If you don't know what that means, look back at maths newsletter 79.
 2. If you get the chance, ask Mr Hamblett to explain to you what a board game is. You may be surprised!
 3. Here's a short demonstration: <https://www.youtube.com/watch?v=6RxKwyFKtmY>

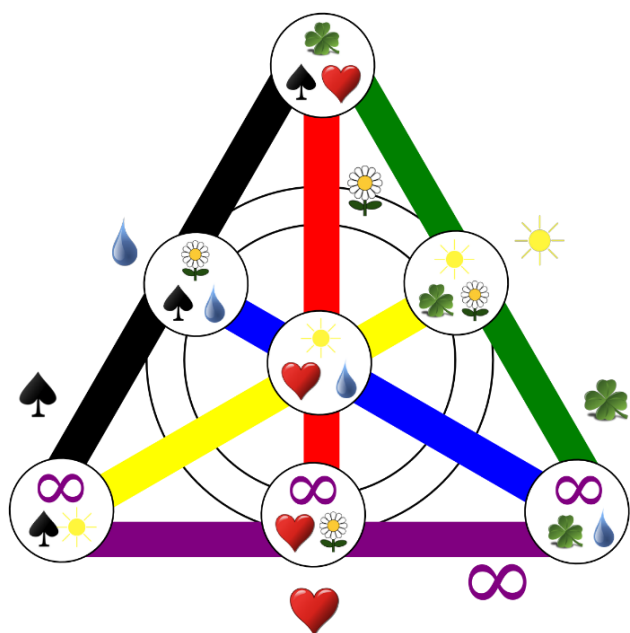
DIY Dobble

Let's have a go at designing our own set of Dobble cards. Remember, every pair of cards must have exactly one symbol in common. Here's one way to do it:



Choose one symbol (sun) and make sure it appears on every card in your deck. You can then fill up the rest of each card using unique symbols. This isn't going to be a very interesting game though.

A better solution uses projective planes. This diagram shows how to use the Fano plane to design a 7 card Dobble game which uses only 7 symbols. Each line corresponds to a different symbol, for example, every card on the blue line contains a raindrop. Why is it important that each pair of cards lie on exactly one line in the plane?



Missing Cards

You can construct a full size Dobble game using a projective plane of order 7.

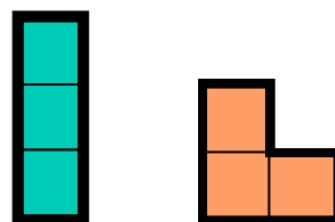
There's an excellent video which explains how to do this⁴. But this makes

$$n^2 + n + 1 = 7^2 + 7 + 1 = 57 \text{ cards}$$

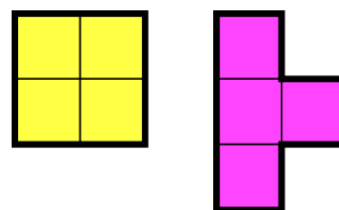
A standard Dobble game only has 55. Where did the two missing cards go? No one really knows. You could use a rainy afternoon to find out which cards are missing and add them to your deck.

Puzzle

Polyominoes are shapes made by joining together squares. There are two different triominoes you can make using three squares (shapes which are identical by rotation or reflection are not counted).



There are five tetrominoes (made from four squares). Can you find the other three?



How many different pentominoes can you find?⁵

Famous Conjecture

Projective planes of order n exist whenever n is a power of a prime number. Mathematicians suspect these are the only projective planes, but it remains an important unsolved problem.

4. https://www.youtube.com/watch?v=VTDKqW_GLkw. You could even follow the instructions to produce your own game of Dobble.

5. If you want to see how many hexominoes there are, look back at maths newsletter 35.