King Edward VI Camp Hill School for Girls

Maths Department Newsletter

19th July 2021

M_{89} is the 10th Mersenne prime

News

So I thought we should have one more maths newsletter before we break up for the summer. Talking of summers, sometimes the sum of a sequence of numbers can be very surprising. For example, did you know that if you add the never-ending series of fractions

$$\sum_{n=1}^{\infty} \frac{1}{n^s}$$

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\ldots$$

they add up to infinity? Well, of course they do, you might think, because there are infinite number of fractions. Of course they will add up to infinity! But it's not that easy. It just so happens that

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots=2.$$

There we have an infinite number of fractions that add up to a finite sum. If that wasn't bad enough[1], the infinite series

$$1-1+1-1+1-1+1-1+1-\ldots.$$

adds up to different things depending on how you do it. If you add them like this:

$$(1-1)+(1-1)+\ldots=0+0+\ldots=0$$

but if you add them like this:

$$1\,(-1+1)\,(-1+1)+\ldots=1+0+0+\ldots=1$$

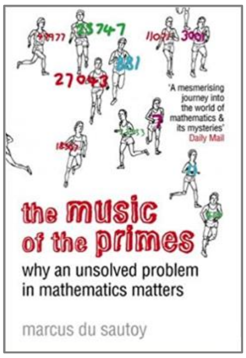
But the weirdest one of all has to be Ramanujan's series

$$1+2+3+4+5+6+7+\ldots=-\frac{1}{12}$$

Look this up if you don't believe me![2]

A Famous Sum

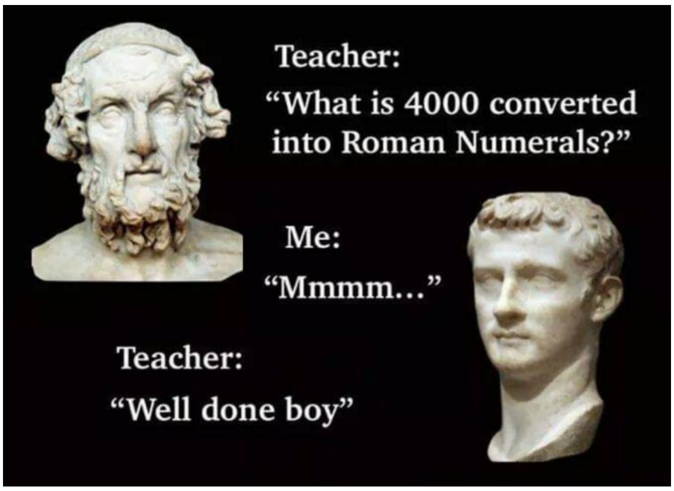
The most famous unsolved maths problem has to do with finding the sum of a series. It's called the **Riemann Hypothesis** and has to do with something called the **Riemann Zeta Function**. This is an infinite sum (this one) that has to do with prime numbers. If you let $s = 1$ you get the **harmonic series** (this one). If you let $s = 2$, you get the series



$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\ldots$$

which the great 18th century maths genius **Leonhard Euler** showed was actually equal to $\frac{\pi^2}{6}$.[3] If you think this sounds interesting, why not read the book ***The Music of the Primes*** by Marcus du Sautoy. It's not a difficult book and you would learn lots of interesting facts about maths.

Joke

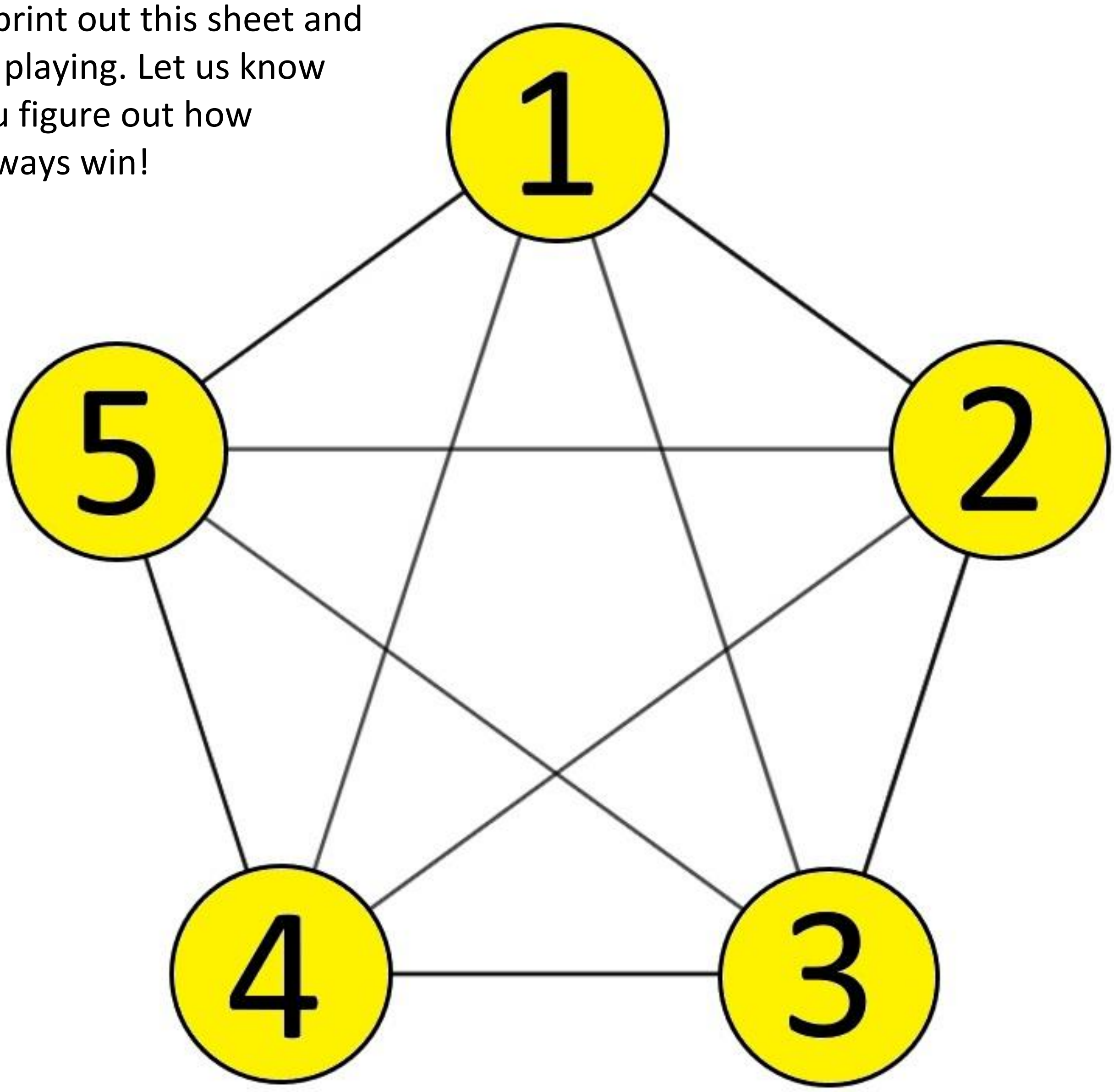


1. It's not bad. It's things like this that make maths really interesting.
2. If you want to find out more about this, watch this: https://www.youtube.com/watch?v=w-I6XTVZXww
3. Can you see what series you get when you let s = -1? You should recognise it.

A Mathematical Game

If you don't like the idea of reading *The Music of the Primes*, but you would still like to do something that involves a bit of mathematical thinking, here's a game you could play. It's a game for two players that just involves adding. To play it you will need something like a coin that you can use as a counter.[4] The first player places the coin[5] onto one of the numbers. This number is added to the running total of the game (which is zero at the start of the game). The two players then take it turns to move the coin onto another number. Each time a player moves the coin, the number of the circle it is moved onto is added to the running total of the game. The aim of the game is to reach a total of 23. The first player to make the score greater than 23 loses the game. The idea then is to see if you can work out a strategy for winning.

Just print out this sheet and start playing. Let us know if you figure out how to always win!



Have a great summer holiday and we'll see you in September!

4. You could even use a counter, but who has those in their house?
5. or counter – I'll just call it a coin to keep things simple.