King Edward VI Camp Hill School for Girls

Maths Department Newsletter

13th October 2023

100 $= 10^2$

News

I did think that perhaps for the 100th maths newsletter, I should do something special, but then it occurred to me that the maths newsletter is always special, so perhaps I should carry on as normal. That seemed to me like a different and better way of looking at the things; and this, when you think about it, is what maths is all about.

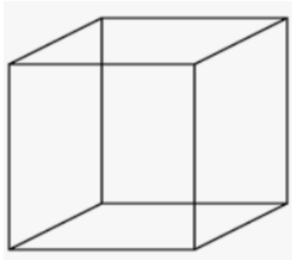
It's basically about different ways of looking at things. Like the famous optical illusion *All is Vanity* pictured above. Is it a girl sitting in front of a mirror or is it a human skull?[1] Sometimes you can look at the same thing in two different ways and see two different things. A classic example of this is the Necker Cube (see above right). On other occasions, though, you may be looking at two different things and think that they are the same thing. For example, you may think that 3 × 4 and 4 × 3 are the same thing, but they are not. Here is a joke about this kind of thing.

Joke

Waiter: Would you like your pizza cut into six slices or eight?
Customer: Six, please. I could never eat eight.

The Necker Cube

One of the best known examples of something that you can look at in two different ways is the **Necker Cube**. This is an optical illusion that was first published in 1832. Does the cube point upwards and to the right, or down and to the left? As you look at it, you should be able to make it flip from one version to the other. This is because the drawing has two[2] equally valid interpretations.



Grandi's Series

Talking about things that can be thought about in more than one way, what is the sum of the following infinite series?

$$1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \ldots$$

We know that when we're adding and subtracting, we can put in brackets, so if we think of the series like this:

$$(1 - 1) + (1 - 1) + (1 - 1) + \ldots$$

This is just 0 + 0 + 0 + 0 + … which adds up to 0, doesn't it? But what if we do this?

$$1 + (-1 + 1) + (-1 + 1) + \ldots$$

This is just 1 + 0 + 0 + 0 + 0 + 0 + … which adds up to 1… doesn't it? There is also a way to make the series add up to 0.5, but I don't have enough space here to show you how to do it. If you're interested, follow the link in the footnote.[3]

1. A human skull with a girl's face on it. I don't know why a skull would have a face on it either. Anyway, it's a good painting. Stop being awkward.
2. I should probably say 'at least two'.
3. https://en.wikipedia.org/wiki/Grandi%27s_series

Lagrange's Four Square Theorem (part 3)

So far, we have been looking at how it's sometimes useful to look at something in different ways. If you're in year 10 or above, this should not come as a surprise to you because you have already come across the following example. Let's say you have been asked to sketch the quadratic function

$$y = x^2 - 6x + 8$$

You know that if the function is written in this form, you can easily see that the y-intercept will be at (0, 8) because 8 is the constant term.

If you want to find the x-intercepts, though, it is better to have the function written in its factorised form. If you look at the function

$$y = (x - 2)(x - 4)$$

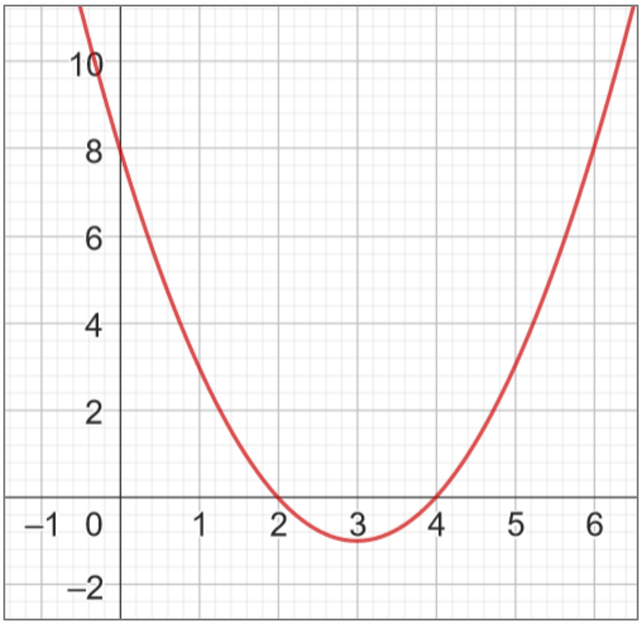
you can see that the x-intercepts will be (2, 0) and (4, 0) because 2 and 4 are the values of x that will make one of the brackets (and therefore the value of the function) equal to zero.

If you want to find the turning point of the curve, though, it is best to have the function written in completed square form.

$$y = (x - 3)^2 - 1$$

From this you can see that the turning point will be at (3, –1) because the lowest value that y can be is –1, and this will occur when $x = 3$. It is important to remember, though, that all these functions are the same function. It is just that it's possible to look at the function in several different ways.

Looking at the same thing in different ways is going to be an idea that we keep coming back to while proving Lagrange's Four Square Theorem, so it would be useful if you keep it in mind. By the way, here is what the sketch of $y = x^2 - 6x + 8$ looks like.



Puzzle

Find two numbers, neither of which contain any zeroes, that multiply together to make 10000.[4]

Competition

The annual Southampton University Cipher Challenge has now started. Fortunately, if you want to take part in this but haven't yet signed up on their website, the rounds that you get points for don't start until the beginning of November. There are practice rounds already live on the site though, so it would probably be best to register your team as soon as possible and get started![5] Don't forget, this is a competition we have come close to winning in the past! ☺

4. The trick to working this out is to think about the prime factorisation of 10000.
5. Do that here: https://www.cipherchallenge.org/account-login/