King Edward VI Camp Hill School for Girls

Maths Department Newsletter

6th October 2023

99 $= 2^3 + 3^3 + 4^3$

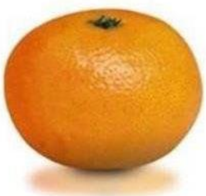
News

Year 9 have been studying trigonometry recent – actually, so have year 11, year 12 and year 13 – but somebody in year 9 asked me this week why the tan (short for tangent) ratio is called the tangent ratio, and I thought that perhaps this is something I should talk about in the maths newsletter.

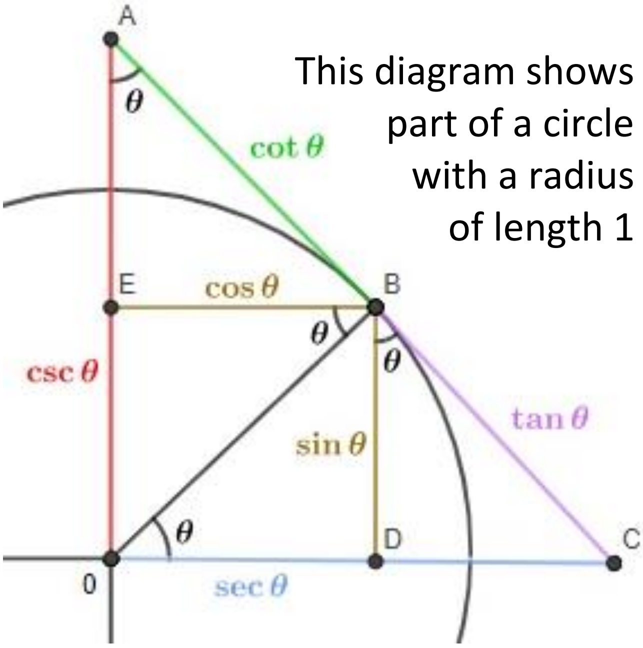
Trigonometry has a long and complicated history, but it is generally considered to have been invented (or is it discovered?[1]) by the Greek mathematician Hipparchus in the 2nd century BC[2]. The tangent ratio gets its name from the fact that, as you can see from the diagram, the purple line labelled tan θ is a tangent to the circle. The other part of that tangent (that joins point B to the vertical axis) is called the cotan (or cot for short). The secant (sec θ) gets its name from the Latin word *secans*, which means 'cutting', as the line cuts through the circle. The line that cuts through the circle in the other direction is called the cosecant.

Joke

$$\frac{\sin(\text{gerine})}{\cos(\text{gerine})} =$$



Trigonometry



This diagram shows part of a circle with a radius of length 1

This diagram shows how the six main trigonometric ratios are related to a unit circle. A **unit circle** is what mathematicians call a circle with a radius of length 1 unit.

Puzzle

For GCSE Maths, you only need to know about sin, cos and tan but at A-level you learn these three new ones:

$$\csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

Can you work out, using the triangle OBD and the facts you know about sin, cos and tan, why the lines in the diagram labelled with the six different ratios are actually equal to those ratios? You will need to look at the various different triangles in the diagram and which ones are similar. Let us know if you get stuck and need any help!

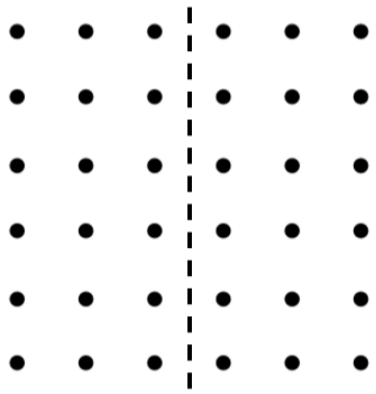
1. Whether mathematical ideas are invented or discovered is something that mathematicians discuss a lot.
2. That's him in the picture. He just looks like every other bearded Greek philosopher/mathematician. I think it must have been a popular look back then.

Lagrange's Four Square Theorem (part 3)

Starting from two newsletters ago (number 97), I've been exploring the ideas needed to prove Lagrange's Four Square Theorem. Last time we thought about what happens when you add together odd and even numbers[3], so now we're going to think about what happens when you square them.

If we multiply a number by itself, the rectangle that it forms becomes a square, which is why multiplying a number by itself is called squaring the number.

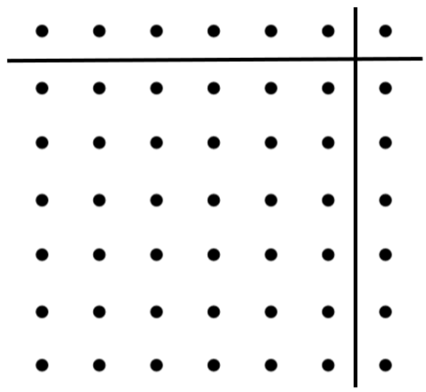
If we square an even number, like 6 for example, it makes a square like this:



Because the length of each side is an even number, we are able to divide the square down the middle into two equal halves.[4] Remember that in newsletter 97 we defined even numbers as "numbers that can be divided exactly into two equal parts which are also whole numbers". Well, this is clearly what we will always have when we make a square whose side length is an even number, so we can say with confidence that

The square of any even number is an even number

So what about when we square an odd number? Let's think, for example, about what happens when we square the number 7.



Because an odd number is just an even number plus one, we can divide up the square as in the diagram. The big square is the square of an even number, which we know is an even number. Around the edge we have an even number of dots along the top of the big square and an even number of dots down the side of the big square. So far then, that's three even numbers. If we add three even numbers together we get another even number. We also, however, get that one dot, on its own, in the corner of the square, and when we add that one dot to the total so far, we get an even number plus one, which is the definition of an odd number, so we also have the rule:

The square of any odd number is an odd number

Apology

Dr Kerr pointed out to me that in the last maths newsletter, I mistakenly described Julius Caesar as a Roman emperor, when actually he was merely a dictator. We apologise for this error.[5]

3. If you didn't read the back of the previous two newsletters, it might be a good idea to do that.
4. Yes, I know halves are always equal by definition. Stop being so pedantic.
5. Mistakenly thinking Julius Caesar was a Roman emperor is a classic error… as in, you know, 'classics'… sorry…