King Edward VI Camp Hill School for Girls

Maths Department Newsletter

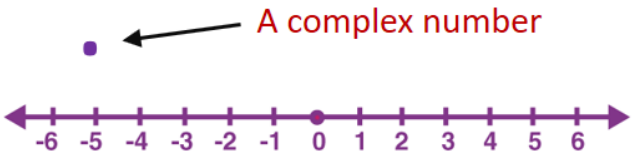
14th December 2021

The 8 queens problem has 92 solutions

News

There are lots of articles on the internet proving that Father Christmas exists and is real[1], but are those two things necessarily the same thing? In maths, there are numbers that exist but are not real. These are called **complex numbers**, and if you want to know more about them you will need to take A-level Further Maths in year 12, as they are not mentioned in GCSE Maths. All the numbers you meet at GCSE are located somewhere on the number line, but complex numbers are located in the space that surrounds the number line.

A random snowman



If this seems weird, that's because it is, but is it really any more weird than negative numbers? Do negative numbers really exist? Have you ever had a negative amount of anything?[2] Negative numbers may be useful for talking about things like temperatures, and debts that you owe the bank because you spent too much money on Christmas presents, but there really isn't any such thing as a negative temperature, or a negative amount of money, is there? Or is there? What do you think? Do negative numbers really exist? Let us know your thoughts on this.

Maths Words

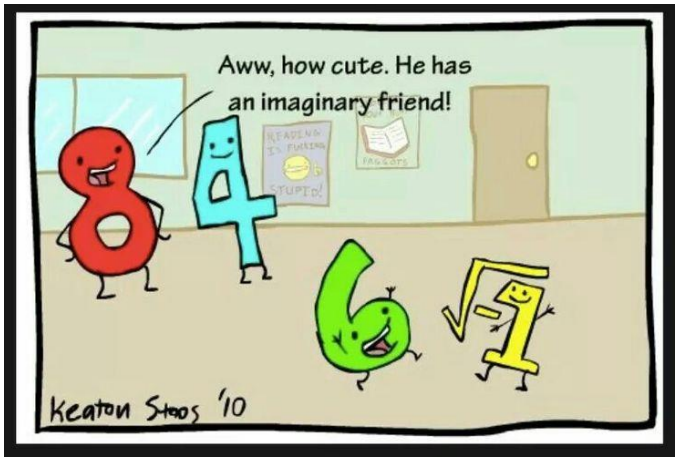
An **imaginary number** is the square root of a negative number. If you try to square root a negative number on your calculator, you will probably get an error. That's because your calculator has not been programmed to have an imagination. Mathematicians would say that the square root of –1 is i. In other words[3]

$$i^2 = -1$$

Similarly, the square root of –4 is $2i$, and the square root of –9 is $3i$, and so on. A **complex number** is what you get when you add together a real number and an imaginary number. To be fair, that does sound like a complex thing to do, doesn't it? Complex numbers are fascinating and they lead to all sorts of interesting things such as Euler's famous identity

$$e^{i\pi} + 1 = 0$$

Joke



1. We did this in newsletter 53, so I'm not going to do it again here.
2. You haven't.
3. Obviously, by 'words' I actually mean symbols.

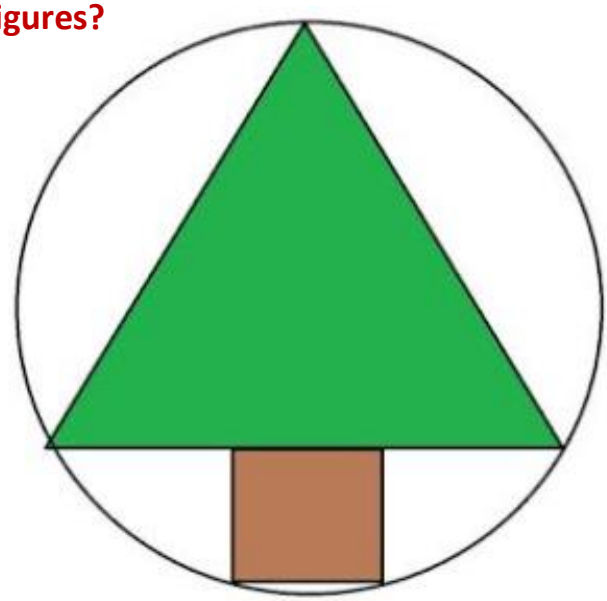
Christmas Puzzle

Mrs Bennett has a box of ornaments that she uses to decorate her tree with each year. All but six of them are blue, all but six of them are red, and all but six of them are green. How many ornaments of each colour does she have?

Ritangle Puzzle

Here is a more difficult puzzle. This is a question from the recent 'Ritangle 2021' competition that some of year 13 have been working on for the past few weeks.

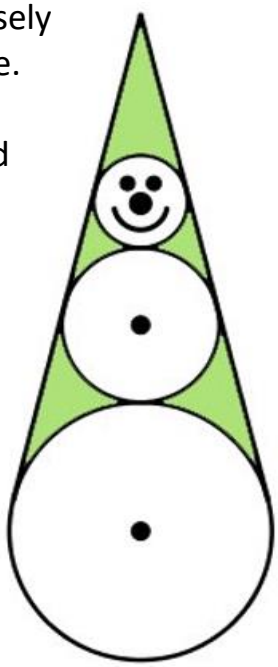
A Christmas tree consists of a green equilateral triangle and a brown square inside a circle of radius 1 as shown. What is the area of the tree, to 3 significant figures?



Because this is a relatively difficult question, we'll tell you that the answer is 1.52 to 3 significant figures, but can you work out why it is this? Use your knowledge of angles, and maybe simple circle equations if necessary.[4]

Another Puzzle

Here's another very loosely Christmas-related puzzle.



Three circles are stacked on top of each other, so that all three of them share two tangents in common that meet at a point above the stack of three circles.[5]

The radii of the three circles are r_1, r_2 and r_3, where r_1 is the radius of the smallest circle – the one with the face.

What is the value of r_1?

$$r_3 = 20\,\text{cm}$$
$$r_2 = 12\,\text{cm}$$
$$r_1 = ?$$

Square Roots

Talking about square roots[6], I recently saw this method for estimating the value of a square root.

$$\sqrt{X} \approx \frac{X+Y}{2\sqrt{Y}}$$

Y= The nearest number to have an integer square root

$$\sqrt{23} \approx \frac{23+25}{2\sqrt{25}} = 4.8$$

$$\sqrt{50} \approx \frac{50+49}{2\sqrt{49}} = 7.071$$

What do you think of this? Is it really clever, or is it just kind of stating the obvious? Or is it both? Anyway, we hope you have great Christmas!

4. You may not need to use the equation of a circle to get the area of the brown square, but that's how I did it.
5. I've basically just described the diagram there. I suppose I could have just said "Look at the diagram."
6. I was. Honestly. Remember, on the other side of the sheet when I was talking about imaginary numbers.