King Edward VI Camp Hill School for Girls





Maths Department Newsletter

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News

Back in the early 1980s, when nobody had heard of storing music on compact discs,

it was decided that a CD should be able to store 74 minutes worth of music, because that's how long Beethoven's 9th Symphony is, and they wanted to make



sure that they could fit the whole symphony on one disc.¹ But that shouldn't be too surprising, as music and numbers have always been very closely related. For example, think about time signatures.



The number 3 here tells you that there are 3 beats in a bar (a bar, as you know, is a musical unit of measure). The 4 tells you the length of each of these beats. Music written in 6/8 time sounds similar to music in 3/4 time, except that the beats are shorter, so you get two sets of three beats per bar instead of one. The fact that 6/8 time is similar to 3/4 time is related to the fact that, mathematically, three quarters is an equivalent fraction to six eighths.

This is why the mathematician and philosopher Gottfried Leibniz once said that "Music is the pleasure the human mind experiences from counting, without being aware that it is counting."

Maths Word

The original compact discs could store 74 minutes of music

In maths, a **ratio** tells you how many times one number contains another number. Here, we can say that the red and green rectangles are in the ratio 3:2, because for every three red rectangles, there are two green rectangles.



If you were to play an open string on a guitar, then you played the same string at the 12th fret, you would get the same note as before, except it would be one octave higher. This is because the 12th fret divides the length of the string exactly in half, which is the same as doubling the frequency of the note.



The frequency of a note and the frequency of a note exactly one octave higher are in the ratio 1:2.

Pythagoras knew about this. As well as knowing that the ratio between two notes an octave apart is 2:1, he also knew that two strings whose lengths were in the ratio 3:2



would produce two notes where the interval between them was a perfect fifth (for example, C and G). Similarly, a ratio of 4:3 produces a perfect fourth (for example, C and F). Two strings in the ratio 9:8 will produce two notes that are 1 tone apart.

2. You can listen to this song here: https://www.youtube.com/watch?v=YBcdt6DsLQA

^{1.} Possibly the strangest thing about this story is that it's almost certainly true. You can find out more about it at https://www.classicfm.com/discover-music/why-is-a-cd-74-minutes/

^{3.} https://www.cnet.com/news/magical-mystery-song-math-solves-beatles-songwriting-puzzle/

The Beatles



For years, there was some dispute over whether John Lennon or Paul McCartney wrote the melody to the Beatles' song *In My Life*.² In a 1976 book, McCartney recalled that it was his work, saying, "I liked *In My Life*. Those were words that John wrote, and I wrote the tune to it. That was a great one." In 1980, however, John Lennon claimed that he had written it. Recently, mathematicians have been able to use statistics to analyse the music and have worked out that there is only a probability of 0.189 that McCartney wrote the melody to the song.³

Juzzle⁴

If it takes one pianist 15 minutes to play the whole of Beethoven's Moonlight Sonata, how long would it take three pianists?

Did You Know?

There are 56 ways to arrange the rhythm of a four-beat bar using only semibreves, minims, crotchets and quavers.

Puzzle

If you take the fact in the 'Did You Know?' and now allow semi-quavers, how many ways will there now be?⁵

Your Favourite Number

Have you ever thought about the fact that when a piece of music has been encoded digitally, either on a compact disc or as a downloadable computer file, it is converted into one big long number? Every digital recording of a piece of music is just one big number. This means that if anyone ever asks you what your favourite number is, you could reply by telling them what your favourite song is.⁶ This also means that some really big numbers (the ones which are pieces of music) are 'musical numbers', although most are not. Could there ever be a mathematical way of identifying musical numbers just by analysing them somehow? Could computers write music just by generating musical numbers?

Einstein

Albert Einstein said about his famous theory of relativity: "The theory of relativity occurred to me by intuition, and music is the driving force behind this intuition. My parents had me study the violin from the time I was six. My new discovery is the result of musical perception." His wife Elsa said, "Music helps him when he is thinking about his theories. He goes to his study, comes back, strikes a few chords on the piano, jots something down, returns to his study."⁷



- 4. Remember, a juzzle is a joke that's more of a puzzle, as opposed to a poke, which is a puzzle that's more of a joke.
- 5. This is a really hard question, so don't spend too long thinking about it!

7. The full article is here: https://www.maths.ox.ac.uk/node/29680

^{6.} You will need to decide for yourself, though, whether you want to be the kind of person who says this kind of thing, and what the implications of saying things like this might be for you socially.