

Chapter 7: INDICES

Basic rules of indices

y^4 means $y \times y \times y \times y$.

4 is called the **index** (plural: indices), **power** or **exponent** of y .

There are 3 basic rules of indices:

- | | | | |
|----|----------------------------|------|------------------------|
| 1) | $a^m \times a^n = a^{m+n}$ | e.g. | $3^4 \times 3^5 = 3^9$ |
| 2) | $a^m \div a^n = a^{m-n}$ | e.g. | $3^8 \times 3^6 = 3^2$ |
| 3) | $(a^m)^n = a^{mn}$ | e.g. | $(3^2)^5 = 3^{10}$ |

Further examples

$$y^4 \times 5y^3 = 5y^7$$

$$4a^3 \times 6a^2 = 24a^5$$

(multiply the numbers and multiply the a 's)

$$2c^2 \times (-3c^6) = -6c^8$$

(multiply the numbers and multiply the c 's)

$$24d^7 \div 3d^2 = \frac{24d^7}{3d^2} = 8d^5$$

(divide the numbers and divide the d terms i.e. by

subtracting

the powers)

Exercise A

Simplify the following:

1) $b \times 5b^5 =$

(Remember that $b = b^1$)

2) $3c^2 \times 2c^5 =$

3) $b^2c \times bc^3 =$

4) $2n^6 \times (-6n^2) =$

5) $8n^8 \div 2n^3 =$

6) $d^{11} \div d^9 =$

7) $(a^3)^2 =$

8) $(-d^4)^3 =$

Help:

<http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=powers/indicesPart2&taskID=1045>

More complex powers

Zero index:

Recall from GCSE that

$$a^0 = 1.$$

This result is true for any non-zero number a .

Therefore $5^0 = 1$ $\left(\frac{3}{4}\right)^0 = 1$ $(-5.2304)^0 = 1$

Negative powers

A power of -1 corresponds to the reciprocal of a number, i.e. $a^{-1} = \frac{1}{a}$

Therefore $5^{-1} = \frac{1}{5}$

$$0.25^{-1} = \frac{1}{0.25} = 4$$

$$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$$

(you find the reciprocal of a fraction by swapping the top and bottom over)

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

This means:

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$$

Fractional powers:

Fractional powers correspond to roots:

$$a^{1/2} = \sqrt{a}$$

$$a^{1/3} = \sqrt[3]{a}$$

$$a^{1/4} = \sqrt[4]{a}$$

In general:

$$a^{1/n} = \sqrt[n]{a}$$

Therefore:

$$8^{1/3} = \sqrt[3]{8} = 2$$

$$25^{1/2} = \sqrt{25} = 5$$

$$10000^{1/4} = \sqrt[4]{10000} = 10$$

A more general fractional power can be dealt with in the following way: $a^{m/n} = (a^{1/n})^m$

So $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$

$$\left(\frac{8}{27}\right)^{2/3} = \left(\left(\frac{8}{27}\right)^{1/3}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{25}{36}\right)^{-3/2} = \left(\frac{36}{25}\right)^{3/2} = \left(\sqrt{\frac{36}{25}}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$$

Help:

<http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=powers/indicesPart3&taskID=1301>

CHALLENGE QUESTIONS

Question 1

Simplify $8 \times 4^3 + 2 \times 4^4$ giving your answer in the form 4^b

.....
(4 marks)

Question 2

Solve the equation

$$9^{(2x^2+2x)} = 27$$

.....

Question 3

Suppose that $8^m = 27$.

What is the value of 4^{m^2} ?

.....

Question 4

Given that $3^{-n} = 0.2$, find the value of $(3^4)^n$

.....
(2 marks)

Question 5

$x^{\frac{1}{2}} = 6$ and $y^{-3} = 64$

Work out the value of $\frac{x}{y}$

$$\frac{x}{y} = \dots\dots\dots$$

(4 marks)

Question 6

Simplify:

$$\frac{b\sqrt{b}}{\sqrt[3]{b}}$$

Leaving you answer in the form:

$$b^n$$

Where n is a fraction to be found.

$n = \dots\dots\dots$

Question 7

Solve $9^{x+4} = 3^{\frac{10}{x}}$.

$\dots\dots\dots$

Question 8

Simplify:

$$\frac{(3x^2y^3)^3 \times (2x^4y^{-2})^4}{8\sqrt[4]{16x^{12}y^8}}$$

$\dots\dots\dots$

Exercise B:

Find the value of:

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(\frac{1}{9}\right)^{1/2}$

4) 5^{-2}

5) 18^0

6) 7^{-1}

7) $27^{2/3}$

8) $\left(\frac{2}{3}\right)^{-2}$

9) $8^{-2/3}$

10) $(0.04)^{1/2}$

11) $\left(\frac{8}{27}\right)^{2/3}$

12) $\left(\frac{1}{16}\right)^{-3/2}$

Simplify each of the following:

13) $2a^{1/2} \times 3a^{5/2}$

14) $x^3 \times x^{-2}$

15) $(x^2y^4)^{1/2}$