Chapter 7: INDICES

Basic rules of indices

 y^4 means $y \times y \times y \times y$.

4 is called the **index** (plural: indices), **power** or **exponent** of y.

There are 3 basic rules of indices:

$$1) a^m \times a^n = a^{m+n}$$

e.g.
$$3^4 \times 3^5 = 3^9$$

$$2) a^m \div a^n = a^{m-n}$$

e.g.
$$3^8 \times 3^6 = 3^2$$

$$(a^m)^n = a^{mn}$$

e.g.
$$(3^2)^5 = 3^{10}$$

Further examples

$$y^4 \times 5 y^3 = 5 y^7$$

$$4a^3 \times 6a^2 = 24a^5$$

(multiply the numbers and multiply the a's)

$$2c^2 \times \left(-3c^6\right) = -6c^8$$

(multiply the numbers and multiply the c's)

the powers)

$$24d^7 \div 3d^2 = \frac{24d^7}{3d^2} = 8d^5$$

(divide the numbers and divide the d terms i.e. by

subtracting

Exercise A

Simplify the following:

1)
$$b \times 5b^5 =$$

(Remember that $b = b^1$)

2)
$$3c^2 \times 2c^5 =$$

$$3) b^2c \times bc^3 =$$

4)
$$2n^6 \times (-6n^2) =$$

5)
$$8n^8 \div 2n^3 =$$

6)
$$d^{11} \div d^9 =$$

$$(a^3)^2 =$$

$$(-d^4)^3 =$$

Help:

http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=powers/indicesPart2&taskID=1045

More complex powers

Zero index:

Recall from GCSE that

$$a^0 = 1$$
.

This result is true for any non-zero number a.

$$5^0 = 1$$

$$\left(\frac{3}{4}\right)^0 = 1$$

$$(-5.2304)^0 = 1$$

Negative powers

A power of -1 corresponds to the reciprocal of a number, i.e. $a^{-1} = \frac{1}{a}$

$$5^{-1} = \frac{1}{5}$$

$$0.25^{-1} = \frac{1}{0.25} = 4$$

$$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$$

(you find the reciprocal of a fraction by swapping the

top and

bottom over)

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

This means:

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$$

Fractional powers:

Fractional powers correspond to roots:

$$a^{1/2} = \sqrt{a}$$

$$a^{1/3} = \sqrt[3]{a}$$

$$a^{1/4} = \sqrt[4]{a}$$

In general:

$$a^{1/n} = \sqrt[n]{a}$$

Therefore:

$$8^{1/3} = \sqrt[3]{8} = 2$$

$$25^{1/2} = \sqrt{25} = 5$$

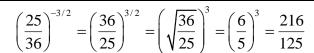
$$10000^{1/4} = \sqrt[4]{10000} = 10$$

A more general fractional power can be dealt with in the following way: $a^{m/n} = (a^{1/n})^m$

So

$$4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$$

$$\left(\frac{8}{27}\right)^{2/3} = \left(\left(\frac{8}{27}\right)^{1/3}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$



Help:

http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=powers/indicesPart3&taskID=1301

CHALLENGE QUESTIONS

Question 1

Simplify $8 \times 4^3 + 2 \times 4^4$ giving your answer in the form 4^b

(4 marks)

Question 2

Solve the equation

$$9^{(2x^2+2x)} = 27$$

Question 3

Suppose that $8^m = 27$.

What is the value of 4^m ?

Question 4

Given that $3^{-n} = 0.2$, find the value of $(3^4)^n$

(2 marks)

Question 5

$$x^{\frac{1}{2}} = 6$$
 and $y^{-3} = 64$
Work out the value of $\frac{x}{y}$

Question 6

Simplify:

$$\frac{b\sqrt{b}}{\sqrt[8]{b}}$$

Leaving you answer in the form:

$$b^n$$

Where n is a fraction to be found.

	$n = \dots$
Ouestion 7	
Question 7 Solve $9^{x+4} = 3^{\frac{10}{x}}$.	
Question 8 Simplify:	
$rac{(3x^2y^3)^3 imes(2x^4y^{-2})^4}{8\sqrt[4]{16x^{12}y^8}}$	
, ,	

Exercise B:

Find the value of:

3)
$$\left(\frac{1}{9}\right)^{1/2}$$

4)
$$5^{-2}$$

8)
$$\left(\frac{2}{3}\right)^{-2}$$

10)
$$(0.04)^{1/2}$$

$$11) \qquad \left(\frac{8}{27}\right)^{2/3}$$

12)
$$\left(\frac{1}{16}\right)^{-3/2}$$

Simplify each of the following:

13)
$$2a^{1/2} \times 3a^{5/2}$$

14)
$$x^3 \times x^{-2}$$

14)
$$x^3 \times x^{-2}$$

15) $(x^2 y^4)^{1/2}$