Chapter 6: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations:
* factorising
* the quadratic formula

Note that not all quadratic equations can be solved by factorising. The quadratic formula can always be used however.

Method 1: Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of x^2 is positive.

Example 1 : Solve $x^2 - 3x + 2 = 0$

Factorise $(x - 1)(x - 2) = 0$
Either $(x - 1) = 0$ or $(x - 2) = 0$
So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

Example 2: Solve $x^2 - 2x = 0$

Factorise: $x(x - 2) = 0$
Either $x = 0$ or $(x - 2) = 0$
So $x = 0$ or $x = 2$

http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=factorising/solveQuadsByFactoring&taskID=1181

Method 2: Using the formula

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3: Solve the equation $2x^2 - 5 = 7 - 3x$

Solution: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$
We can then tell that $a = 2$, $b = 3$ and $c = -12$.
Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the *surd form* for the solutions)

If we have a calculator, we can evaluate these roots to get: $x = 1.81$ or $x = -3.31$

If you need more help with the work in this chapter, you can get help from this web site:

http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=quadraticformula/formulamovie&taskID=1160

EXERCISE

1) Use factorisation to solve the following equations:

a) $x^2 + 3x + 2 = 0$

b) $x^2 - 3x - 4 = 0$

c) $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a) $x^2 + 3x = 0$

b) $x^2 - 4x = 0$

c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a) $6x^2 - 5x - 4 = 0$

b) $8x^2 - 24x + 10 = 0$

4) Use the formula to solve the following equations to 3 significant figures. Some of the equations can't be solved.

a) $x^2 + 7x + 9 = 0$

b) $6 + 3x = 8x^2$

c) $4x^2 - x - 7 = 0$

d) $x^2 - 3x + 18 = 0$

e) $3x^2 + 4x + 4 = 0$

f) $3x^2 = 13x - 16$

CHALLENGE QUESTIONS:

Question 1

I am thinking of a number.

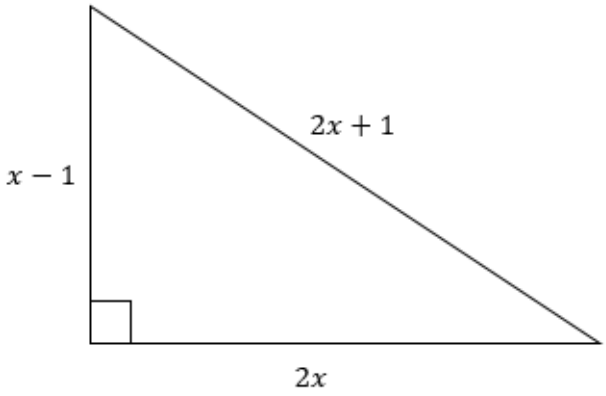
> When I subtract 25 from my number, then square the answer,
>
> I get the **same result as**
>
> when I square my number, then subtract 25 from the answer.

What is my number?

my number is ……

Question 2

By first forming an appropriate equation, determine the value of x.



$x =$ ……

Question 3

The product of two positive integers is equal to twice their sum. The product is also equal to six times the difference between the two integers. What is the sum of these two integers?

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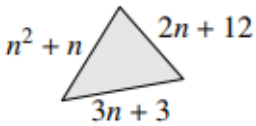
Question 4

The diagram shows a triangle with sides $n^2 + n$, $2n + 12$ and $3n + 3$.



What is the sum of all the values of n for which the triangle is isosceles?

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Question 5

A designer is lining the base and sides of a rectangular drawer with paper.
The width of the drawer is $2x$ cm, the length is $3x$ cm and the height is 5 cm.
The drawer has no top. The total area of paper is 4070 cm^2.

Find the value of x to 1 decimal place, and use this value of x to work out the volume V of the drawer in litres, giving your answer to 1 decimal place.

$x =$ ..... cm

$V =$ ..... litres

Question 6

Alison is using the quadratic formula to solve a quadratic equation.
She substitutes values into the formula and correctly gets

$$x = \frac{-7 \pm \sqrt{49 - 32}}{4}$$

Work out the quadratic equation that Alison is solving.
Give your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

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(3 marks)

Question 7

Solve the equation

$$\frac{1}{x-2} - \frac{1}{x-1} = 2$$

Give your answers to 2 decimal places.

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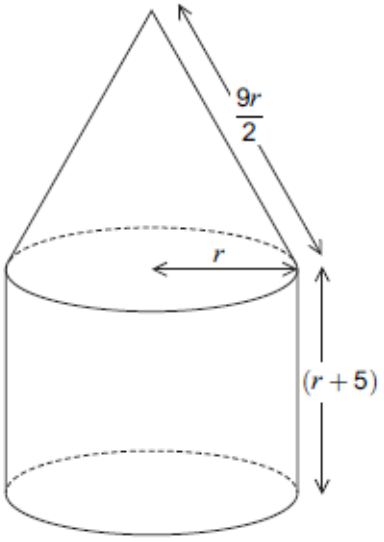
(6 marks)

Question 8

On this diagram all lengths are given in centimetres.
A cylinder and cone are joined together to make a solid.
The cylinder has radius r and height $r + 5$
The cone has radius r and slant height $\frac{9r}{2}$



It can be shown that the total surface area of the solid, in cm^2, is $\frac{5\pi r}{2}(3r + 4)$
The total surface area of the solid is $1200\ picm^{\ 2}$
Work out the value of r.

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