Chapter 4: FACTORISING

Common factors

We can factorise some expressions by taking out a common factor.

Example 1:	Factorise $12x - 30$
Solution: 6	6 is a common factor to both 12 and 30. We can therefore factorise by taking outside a bracket: $12x - 30 = 6(2x - 5)$

Example 2:	Factorise $6x^2 - 2xy$
Solution:	2 is a common factor to both 6 and 2. Both terms also contain an x. So we factorise by taking $2x$ outside a bracket. $6x^2 - 2xy = 2x(3x - y)$

Example 3:	Factorise $9x^3y^2 - 18x^2y$
Solution:	9 is a common factor to both 9 and 18. The highest power of x that is present in both expressions is x^2. There is also a y present in both parts. So we factorise by taking $9x^2y$ outside a bracket: $9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$

Example 4:	Factorise $3x(2x - 1) - 4(2x - 1)$
Solution:	There is a common bracket as a factor. So we factorise by taking $(2x - 1)$ out as a factor. The expression factorises to $(2x - 1)(3x - 4)$

Exercise A

Factorise each of the following

1) $3x + xy$

2) $4x^2 - 2xy$

3) $pq^2 - p^2q$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y - 1) + 3(y - 1)$

Factorising quadratics

Simple quadratics: Factorising quadratics of the form $x^2 + bx + c$
The method is:
<u>Step 1</u>: Form two brackets $(x \ \ldots \ )(x \ \ldots \ )$
<u>Step 2</u>: Find two numbers that multiply to give *c* and add to make *b*. These two numbers get written at the other end of the brackets.

Example 1: Factorise $x^2 - 9x - 10$.

Solution: We need to find two numbers that multiply to make -10 and add to make -9.
These numbers are -10 and 1.
Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

General quadratics: Factorising quadratics of the form $ax^2 + bx + c$
The method is:
<u>Step 1</u>: Find two numbers that multiply together to make *ac* and add to make *b*.
<u>Step 2</u>: Split up the *bx* term using the numbers found in step 1.
<u>Step 3</u>: Factorise the front and back pair of expressions as fully as possible.
<u>Step 4</u>: There should be a common bracket. Take this out as a common factor.

Example 2: Factorise $6x^2 + x - 12$.

Solution: We need to find two numbers that multiply to make $6 \times -12 = -72$ and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = \underbrace{6x^2 - 8x} + \underbrace{9x - 12}$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

Difference of two squares: Factorising quadratics of the form $x^2 - a^2$

Remember that $x^2 - a^2 = (x + a)(x - a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x+3)(x-3)$
$16x^2 - 25 = (2x)^2 - 5^2 = (2x+5)(2x-5)$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x+4)(x-4)$
and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x+4y)(x-4y)$

Factorising by pairing
We can factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing:

$2x^2 + xy - 2x - y = x(2x + y) - 1(2x + y)$ (factorise front and back pairs, ensuring both brackets are identical)

$= (2x + y)(x - 1)$

If you need **more help** with factorising, you can use this website:

http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=factorising/factoriseHigher&taskID=1156

Exercise B

Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$ (factorise by taking out a common factor)

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

13) $4y^3 - 9a^2 y$

14) $8(x+1)^2 - 2(x+1) - 10$

CHALLENGE QUESTIONS

Question 1

Factorise the following fully: 3a(x-2)+6c(2x-4)

....

Question 2

Simplify $\frac{(a+c)(x+1)+3x+3}{2bx+2b}$

....

Question 3

Simplify

$$\frac{x^2 - 8x + 12}{x^2 - 7x + 6}$$

....

Question 4

The nth term of a sequence is $n^2 + 12n + 27$
By factorising, or otherwise, show that the 20th term can be written as the product of two prime numbers.

....
(2 marks)

Question 5

Fully factorise $(3x^2 - x - 6)^2 - (2x^2 - x + 3)^2$

....

Question 6

Show that

$$\frac{1}{2x^2 + x - 15} \div \frac{1}{3x^2 + 9x}$$

simplifies to $\frac{ax}{bx+c}$, where a, b and c are integers to be found.

....
(3 marks)

Question 7

It can be shown that $a^2 - b^2 \equiv (a - b)(a + b)$
Hence, or otherwise, simplify fully $(x^2 + 4)^2 - (x^2 - 2)^2$

....
(3 marks)

Question 8

The teenagers Sam and Jo notice the following facts about their ages:
The difference between the squares of their ages is four times the sum of their ages.
The sum of their ages is eight times the difference between their ages.
What is the age of the older of the two?

.... year-old