

Chapter 5: CHANGING THE SUBJECT OF A FORMULA

We can use algebra to change the subject of a formula. Rearranging a formula is similar to solving an equation – we must do the same to both sides in order to keep the equation balanced.

Example 1: Make x the subject of the formula $y = 4x + 3$.

Solution:

$$y = 4x + 3$$

Subtract 3 from both sides:

$$y - 3 = 4x$$

Divide both sides by 4;

$$\frac{y - 3}{4} = x$$

So $x = \frac{y - 3}{4}$ is the same equation but with x the subject.

Example 2: Make x the subject of $y = 2 - 5x$

Solution: Notice that in this formula the x term is negative.

$$y = 2 - 5x$$

Add $5x$ to both sides

$$y + 5x = 2$$

Subtract y from both sides

$$5x = 2 - y$$

Divide both sides by 5

$$x = \frac{2 - y}{5}$$

(the x term is now positive)

Example 3: The formula $C = \frac{5(F - 32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.

We can rearrange to make F the subject.

$$C = \frac{5(F - 32)}{9}$$

Multiply by 9

$$9C = 5(F - 32)$$

Expand the brackets

$$9C = 5F - 160$$

Add 160 to both sides

$$9C + 160 = 5F$$

Divide both sides by 5

$$\frac{9C + 160}{5} = F$$

Therefore the required rearrangement is $F = \frac{9C + 160}{5}$.

(this removes the fraction)

Exercise A

Make x the subject of each of these formulae:

1) $y = 7x - 1$

2) $y = \frac{x + 5}{4}$

3) $4y = \frac{x}{3} - 2$

4) $y = \frac{4(3x - 5)}{9}$

Rearranging equations involving squares and square roots

Example 4: Make x the subject of $x^2 + y^2 = w^2$

Solution:

$$x^2 + y^2 = w^2$$

Subtract y^2 from both sides:

$$x^2 = w^2 - y^2 \quad (\text{this isolates the term involving } x)$$

Square root both sides:

$$x = \pm\sqrt{w^2 - y^2}$$

Remember that you can have a positive or a negative square root. We cannot simplify the answer any more.

Example 5: Make a the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Solution:

$$t = \frac{1}{4}\sqrt{\frac{5a}{h}}$$

Multiply by 4

$$4t = \sqrt{\frac{5a}{h}}$$

Square both sides

$$16t^2 = \frac{5a}{h}$$

Multiply by h :

$$16t^2h = 5a$$

Divide by 5:

$$\frac{16t^2h}{5} = a$$

Exercise B:

Make t the subject of each of the following

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

More help:

<http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=simplify/rearrangehigher&taskID=1170>

More difficult examples

Sometimes the variable that we wish to make the subject occurs in more than one place in the formula. In these questions, we collect the terms involving this variable on one side of the equation, and we put the other terms on the opposite side.

Example 6: Make t the subject of the formula $a - xt = b + yt$

Solution:

$$a - xt = b + yt$$

Start by collecting all the t terms on the right hand side:

Add xt to both sides:
$$a = b + yt + xt$$

Now put the terms without a t on the left hand side:

Subtract b from both sides:
$$a - b = yt + xt$$

Factorise the RHS:
$$a - b = t(y + x)$$

Divide by $(y + x)$:
$$\frac{a - b}{y + x} = t$$

So the required equation is
$$t = \frac{a - b}{y + x}$$

Example 7: Make W the subject of the formula $T - W = \frac{Wa}{2b}$

Solution: This formula is complicated by the fractional term. We begin by removing the fraction:

Multiply by $2b$:
$$2bT - 2bW = Wa$$

Add $2bW$ to both sides:
$$2bT = Wa + 2bW \quad (\text{this collects the } W\text{'s together})$$

Factorise the RHS:
$$2bT = W(a + 2b)$$

Divide both sides by $a + 2b$:
$$W = \frac{2bT}{a + 2b}$$

Exercise C

Make x the subject of these formulae:

1) $ax + 3 = bx + c$

2) $3(x + a) = k(x - 2)$

3) $y = \frac{2x + 3}{5x - 2}$

4) $\frac{x}{a} = 1 + \frac{x}{b}$

CHALLENGE QUESTIONS:

Question 1

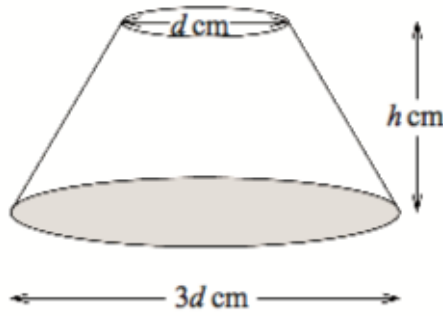


Diagram **NOT** accurately drawn

The diagram shows a frustum.

The diameter of the base is $3d \text{ cm}$ and the diameter of the top is $d \text{ cm}$. The height of the frustum is $h \text{ cm}$.

The formula for the curved surface area, $S \text{ cm}^2$, of the frustum is

$$S = 2\pi d\sqrt{h^2 + d^2}$$

Rearrange the formula to make h the subject.

$h = \dots\dots\dots$

(3 marks)

Question 2

$$y = at^2 - 2at$$
$$x = 2a\sqrt{t}$$

Express y in terms of x and a .

Give your answer in the form

$$y = \frac{x^p}{ma^3} - \frac{x^q}{na}$$

where p, q, m and n are integers.

$y = \dots\dots\dots$

(4 marks)

Question 3

Make q the subject of

$$p = \frac{1}{1 + \frac{1}{1 + \frac{1}{q}}}$$

$q = \dots\dots\dots$

Question 4

Make x the subject of the equation, giving your answer **as a single fraction** (and no fractions within fractions):

$$\frac{1}{1 - \sqrt{y}} = 1 - \frac{1}{x}$$

.....
x =

Question 5

Make x the subject of the following, **fully simplifying your answer**.

$$y + 2 = \frac{x + 1}{x - y}$$

(Note that the left-hand-side of the equation is already written for you)

.....
x =

Question 6

Make y the subject of

$$\frac{y}{x} + \frac{2y}{x + 4} = 3$$

Give your answer as simply as possible.

.....
y =
(5 marks)

