Answers to AS spreads

Units and dimensions

Pages 6–7

1

Physical quantity	Equation used to derive unit	Unit	Symbol and name (if there is one)
frequency	period^{-1}	s^{-1}	Hz hertz
volume	length^3	m^3	–
acceleration	velocity ÷ time	$m\,s^{-2}$	–
force	mass × acceleration	$kg\,m\,s^{-2}$	N newton
work and energy	force × distance	N m (or $kg\,m^2\,s^{-2}$)	J joule
voltage	energy ÷ electric charge	$J\,C^{-1}$	V volt
electrical resistance	voltage ÷ electric current	$V\,A^{-1}$	Ω ohm

2 LHS: C = A s
RHS: A s

3 LHS: $W = J\,s^{-1} = N\,m\,s^{-1} = kg\,m^2\,s^{-3}$
RHS: $A \times V = A \times J\,C^{-1} = A \times kg\,m^2\,s^{-2} \times (A\,s)^{-1} = A \times kg\,m^2\,s^{-2} \times A^{-1} \times s^{-1} = kg\,m^2\,s^{-3}$

4 $N\,kg^{-1} = kg\,m\,s^{-2} \times kg^{-1} = m\,s^{-2}$

5 LHS: N
RHS: (units of G) $\times \dfrac{kg \times kg}{m^2}$ = (units of G) $\times kg^2 \times m^{-2}$
(Units of G) $= N\,kg^{-2}\,m^2 = kg\,m\,s^{-2} \times kg^{-2}\,m^2 = kg^{-3}\,m^{-1}\,s^{-2}$

Stretch yourself

LHS: s

RHS: $\dfrac{m^{\frac{1}{2}}}{(m\,s^{-2})^{\frac{1}{2}}} = \dfrac{m^{\frac{1}{2}}}{m^{\frac{1}{2}}\,s^{-1}} = \dfrac{1}{s^{-1}} = s$

Uncertainties and significant figures

Pages 8–9

1 **a** 5.7 cm ± 2%
b 2.0 A ± 5%
c 450 kg ± 0.4%
d 10.6 s ± 0.5%
e 47.5 mV ± 1%
f 366 000 J ± 0.3%

2 **a** 1200 ± 120 W
b 34.1 ± 0.34 m
c 330 000 ± 1650 Ω
d 0.00800 ± 0.00008 m

3 **D** 1400 ± 5 mm (Did you calculate them all? The same absolute error means the percentage error will be smallest in the largest measurement, so no need to calculate.)

4 **a** 3 **b** 2
c 4 **d** 4
e 4 **f** 1
g 2 **h** 2
i 2

5 **a** 19 m **b** 120 km
c 21 s **d** 6.6×10^{-34} J s
e 1.7×10^{-27} kg **f** 5.0 s

6 $I = 12\,V \div 3300\,\Omega = 3.636 \times 10^{-3}\,A = 3.6\,mA$

7 $v = 540\,m \div 16\,s = 33.75\,m\,s^{-1} = 34\,m\,s^{-1}$

8 Circumference $C = 2\pi r = 2\pi \times 1.4$ cm = 8.796 cm Uncertainty = (0.1 × 100%)/1.4 = 7%
(7 × 8.796 cm)/100 = 0.6 cm so C = 8.8 ± 0.6 cm
Area $A = \pi r^2 = \pi \times (1.4\,cm)^2 = 6.16\,cm^2$ Uncertainty = 2 × (0.1 × 100%)/1.4 = 14% (because the radius is used twice and this will double the error)
$(14 \times 6.16\,cm^2)/100 = 0.9\,cm^2$ so $A = 6.2 \pm 0.9\,cm^2$

Scale of the Universe

Pages 10–11

1 **a** 1.35×10^3 W (or 1.350×10^3 W to 4 s.f.) **b** 5.03×10^2 N
c 1.3×10^5 Pa **d** 8.64×10^4 s
e 6.96×10^8 s **f** 9.315×10^8 eV
g 1.76×10^{11} C kg^{-1}

2 The Big Bang occurred $\sim 10^{10}$ years ago

3 Mercury, Venus, Earth and Mars $\sim 10^6$ m Jupiter, Saturn, Uranus and Neptune $\sim 10^7$ m

4 **a** 3.0×10^8 m s^{-1} ÷ 3.0×10^{-7} m = 1.0×10^{15} Hz
b 3.0×10^8 m s^{-1} ÷ 1000 m = 3.0×10^5 Hz
c 3.0×10^8 m s^{-1} ÷ 1.0×10^{-10} m = 3.0×10^{18} Hz

5 **a** 2.5×10^{-3} m **b** 6.0×10^{-1} kg
c 1.60×10^{-15} m (or 1.6×10^{-15} m) **d** 1×10^{-8} J
e 5×10^3 m **f** 9.11×10^{-31} kg
g 6.2×10^{-1} N

6 The charge on the electron $\sim 10^{-19}$ C

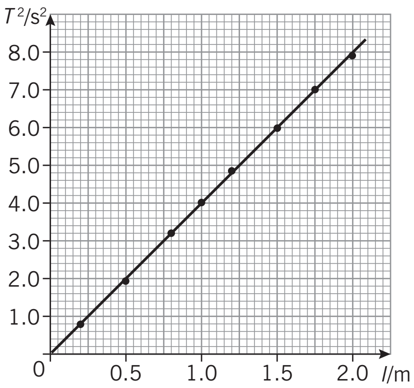
7 Note that these are possible answers, you may have chosen a different prefix:
1 **a** 1.35 × kW (or 1.350 × kW to 4 s.f.)
b 0.503 kN
c 130 kPa
f 931.5 MeV
g 1.76 TC kg^{-1}
5 **a** 2.5 mm
d 0.01 μJ
e 5 km

A practical activity: The pendulum

Pages 12–13

1

l/cm	t/s	T/s	T^2/s^2	l/m
20.0	18	0.90	0.81	0.20
50.0	28	1.40	1.96	0.50
80.0	36	1.80	3.24	0.80
100.0	40	2.00	4.00	1.00
120.0	44	2.20	4.84	1.20
150.0	49	2.45	6.00	1.50
175.0	53	2.65	7.02	1.75
200.0	56	2.80	7.84	2.00



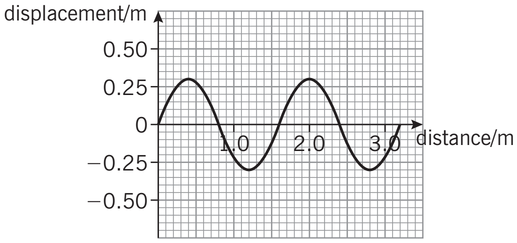
2 The graph is a straight line through the origin showing $T^2 \propto l$. This agrees with the equation $T^2 = 4\pi^2 (l/g)$ if the gradient $= 4\pi^2/g$ that is if $g = 4\pi^2$/gradient

$$\text{Gradient} = \frac{(8.0 - 0.0)\,\text{s}^2}{(2.0 - 0.0)\,\text{m}} = 4.00\,\text{s}^2\,\text{m}^{-1}$$

$g = 4\pi^2/4.00$ s^2 m^{-1} = 9.87 m s^{-2} which is very close to the actual value of 9.81 m s^{-2}

Graphs of waves

Pages 14–15

1


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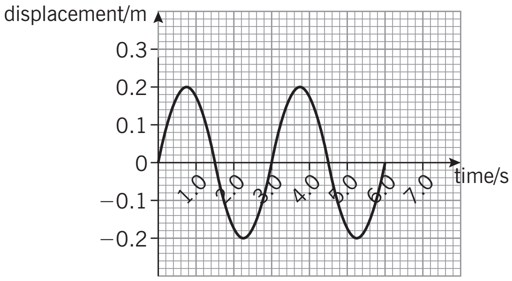


3 $v = 300\ \text{Hz} \times 1.5\,\text{m} = 450\,\text{m}\,\text{s}^{-1}$

4 $v = 300\,\text{Hz} \times 2.1\,\text{m} = 630\,\text{m}\,\text{s}^{-1}$

Superposition of waves

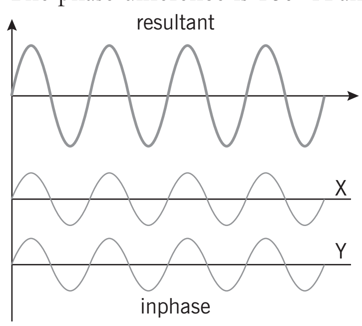
Pages 16–17

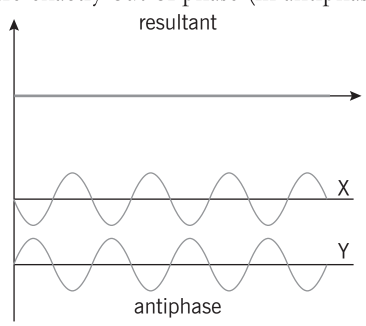
1 **a** The phase difference is 0° (or 360°) X and Y are in phase

b The phase difference is 90° (X is 90° ahead of Y)

c The phase difference is 90° (X is 90° behind Y)

d The phase difference is 180° X and Y are exactly out of phase (in antiphase).

2 **a**


b


3 **a** $2L = 1.2\,\text{m}$

b $L = 0.60\,\text{m}$

c The 3rd harmonic will have two nodes between the ends and two nodes at the ends so the string is divided into three vibrating parts: $\lambda = (2/3)\,L = 2 \times 0.60\,\text{m} \div 3 = 0.40\,\text{m}$.

Diffraction and interference

Pages 18–19

Note that if *n* is an integer and you are using a value of 2, say, this does not mean your answer has to be given to 1 s.f. Because it is an integer, you know it is correct to at least the same number of significant figures as the other data you have been given.

1 $\dfrac{8.4 \times 10^{-4}\,\text{m} \times 1.6 \times 10^{-3}\,\text{m}}{2.5\,\text{m}} = 5.3 \times 10^{-7}\,\text{m}$

2 **a** $\dfrac{650 \times 10^{-9}\text{m} \times 2.5\,\text{m}}{1.6 \times 10^{-3}\,\text{m}} = 1.0 \times 10^{-3}\,\text{m}$

b $\dfrac{470 \times 10^{-9}\text{m} \times 2.5\,\text{m}}{1.6 \times 10^{-3}\,\text{m}} = 7.3 \times 10^{-4}\,\text{m}$

3 $\dfrac{1.0 \times 10^{-3}\,\text{m} \sin 19°}{550} = 1 \times \lambda \quad \lambda = 5.9 \times 10^{-7}\,\text{m} = 590\,\text{nm}$

4 $\sin \theta = \dfrac{710 \times 10^{-9}}{1.0 \times 10^{3} \times 10^{3}} = 0.71 \theta = 45°$

Refraction of light

Pages 20–21

1 $n = \frac{2.9979 \times 10^8}{2.2540 \times 10^8} \text{m s}^{-1} = 1.331$

2 $c_{\text{glass}} = \frac{2.9979 \times 10^8 \text{m s}^{-1}}{1.52} = 1.97 \times 10^8 \text{m s}^{-1}$ (n has only 3 s.f., so answer should be to no more than 3 s.f.)

3 water to glass $n = \frac{n_g}{n_w} = 1.52 \div 1.33 = 1.1$

4 $n_e = \frac{1.52 \sin 23°}{\sin 26°} = 1.35 = 1.4$ (2 s.f.)

5 $1.0 \sin 20° = n_g \sin 11°$ $n_g = 1.0 \times 0.342 \div 0.191 = 1.8$

6 $\sin \theta_c = \frac{1.0}{2.4} = 0.417$ $\theta_c = 24.6°$

7 $n_{CZ} \sin 27° = 1.0$ $n_{CZ} = 1 \div \sin 27° = 2.2$

9 Motion 1

Pages 22–23

1 Scalars: density, electric charge, electrical resistance, energy, frequency, mass, power, voltage, volume, work done
Vectors: field strength, force, friction, momentum, weight

2 Scalars: 3 m s^{-1}, 50 km,
Vectors: +20 m s^{-1}, 100 m NE, −5 cm, 10 km S 30°W.

Stretch yourself

Work is a scalar – it has magnitude only. The distance is in the same direction as the force.
Moments are vectors. They are turning forces clockwise or anticlockwise. The distance is perpendicular to the force.
The units are not the same. (You should use J for work and energy and not Nm to avoid confusion.)

3 **a** $(30.0 - 0.0)\text{ cm} \div (3.60 - 0.0)\text{ s} = 8.75 \text{ cm s}^{-1}$
b $(-7.0 - 25.0) \div (7.20 - 5.6)\text{ s} = -20.0 \text{ cm s}^{-1}$

4 **a** $(7.6 - 0.0)\text{ m s}^{-1} \div (8.2 - 0.0)\text{ s} = 0.9 \text{ m s}^{-2}$
b $(0.0 - 12.0)\text{ m s}^{-1} \div (7.8 - 2.7)\text{ s} = -2.4 \text{ m s}^{-2}$

Motion 2

Pages 24–25

1 $0.5 \times (5.2 + 2.8)\text{ s} \times 8.0 \text{ m s}^{-1} = 32 \text{ m}$

2 $0.5 \times 7.0\text{ s} \times 2.4 \text{ m s}^{-1} + (14.0 - 7.0)\text{ s} \times 2.4 \text{ m s}^{-1} + 0.5 \times (16.0 - 14.0)\text{ s} \times 2.4 \text{ m s}^{-1} = 27.6 \text{ m} = 28 \text{ m}$ (2 s.f.)

3 One possible method: area = triangle + trapezium + rectangle + counting squares under curve
$= (0.5 \times 1.4\text{ s} \times 1.2 \text{ m s}^{-1}) + 0.5 \times (1.2 + 8.0)\text{ m s}^{-1} \times (3.6 - 1.2)\text{ s} + (8.0 \text{ m s}^{-1} \times 1.4\text{ s}) + 110 \times (0.4 \text{ m s}^{-1} \times 0.2\text{ s})$
$= 0.84 \text{ m} + 11.88 \text{ m} + 11.2 \text{ m} + 8.8 \text{ m} = 32.72 \text{ m} = 33 \text{ m}$ (2 s.f.)

4 1st graph: 0 (− constant velocity so acceleration = 0)
2nd graph: $2.4 \text{ m s}^{-1} \div 7.0\text{ s} = 0.34 \text{ m s}^{-2}$
3rd graph: gradient of tangent = $[25.0 - (-10.0)\text{ m s}^{-1}] \div 10.0\text{ s} = 3.5 \text{ m s}^{-2}$

Motion 3

Pages 26–27

1 $v = u + at$ $v = (13 \text{ m s}^{-1}) + (4.0 \text{ m s}^{-2})(9.0\text{ s})$ $v = 49 \text{ m s}^{-1}$

2 $v^2 = u^2 + 2as$ $0 = (28 \text{ m s}^{-1})^2 + 2a(75 \text{ m})$ $a = \frac{-784 \text{ m s}^{-2}}{150} = 5.2 \text{ m s}^{-2}$

3 $s = ut + \frac{1}{2}at^2$ $s = (0)(2.9\text{ s}) + \frac{1}{2}(-9.81 \text{ m s}^{-2})(2.9\text{ s})^2$ $s = -41 \text{ m}$ (41 m downwards)

4 ↑ time to hit beach: $s = ut + \frac{1}{2}at^2$ $(-200 \text{ m}) = (0)t + \frac{1}{2}(-9.81 \text{ m s}^{-2})t^2$ $t^2 = \frac{400}{9.81}$ $t = 6.39$ s
→ $s = ut + \frac{1}{2}at^2$ $s_h = (3.0 \text{ m s}^{-1})(6.39\text{s}) + \frac{1}{2}(0)(6.39\text{ s})^2 = 19.2 \text{ m} = 19 \text{ m}$ (2 s.f.)

5 → $s = ut + \frac{1}{2}at^2$ $(0.10 \text{ m}) = (0.50 \text{ km s}^{-1})t + \frac{1}{2}(0)t^2$ $t = \frac{(0.10 \text{ m})}{(0.50 \times 10^3 \text{ m s}^{-1})} = 2 \times 10^{-4}$ s
↑ $s = ut + \frac{1}{2}at^2$ $s = (0)(2 \times 10^{-4}\text{ s}) + \frac{1}{2}(-9.81)(2 \times 10^{-4}\text{ s})^2$ $s = 2.8 \times 10^{-7}$ m

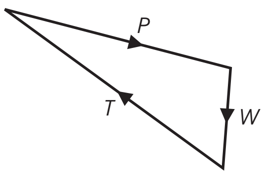
Stretch yourself

↑ $v^2 = u^2 + 2as$ $v_v^2 = (0) + 2(-9.81 \text{ m s}^{-2})(-200 \text{ m})$ $v_v^2 = 3924$ $v_v = -63 \text{ m s}^{-1}$ (downwards)
→ $v = u + at$ $v_h = 3.0 \text{ m s}^{-1}$
Resultant $v = \sqrt{(63^2 + 3.0^2)} = 63 \text{ m s}^{-1}$ angle to horizontal $\tan^{-1}(63 \div 3.0) = 87°$

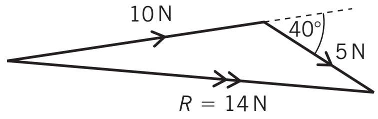
Forces

Pages 28–29

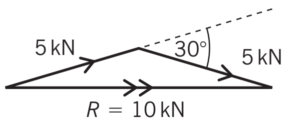
1



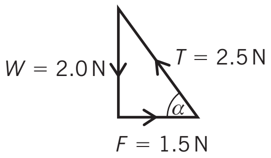
2 **a**



b



3



4 **a** $R = \sqrt{[(3.0\,\text{N})^2 + (4.0\,\text{N})^2]} = 5.0\,\text{N}$ at an angle $\tan^{-1}(3.0\,\text{N} \div 4.0\,\text{N}) = 37°$ to the 4.0 N force

b $R = \sqrt{[(5.0\,\text{N})^2 + (12.0\,\text{N})^2]} = 13\text{N}$ at an angle $\tan^{-1}(5.0\,\text{N} \div 12.0\,\text{N}) = 23°$ to the 12.0 N force

Resolving forces

Pages 30–31

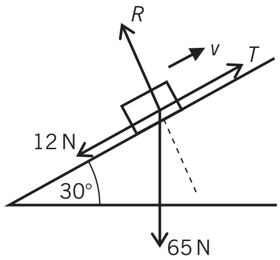
1 Horizontal = 550 N cos 30° = 480 N Vertical = 550 N sin 30° = 275 N

2 $R =$(85 N)cos 15° = 82 N $F =$ (85 N)sin 15° = 22 N

3 75 N = T_1 sin 60° + T_2 sin 30° 0.5 T_2 = 75 N − T_1 × 0.867
T_1 cos 60° = T_2 cos 30° 0.5 T_1 = T_2 cos 30° T_1 = 2 T_2 × 0.867
0.5 T_2 = 75 N − 2 T_2 × 0.867 × 0.867
0.5 T_2 + 1.50 T_2 = 75 N
T_2 = 37.5 N = 38 N T_1 = 2 (37.5 N) × 0.867 = 65 N

4 Resolving for join B: ↑ T_{AB}sin 40° = 3 × 9.81 N → T_{AB}cos 40° = T_{BC}
T_{AB}sin 40° = 29.4N ÷ sin 40° = 45.8N =46N T_{BC} = (45.8 N)cos 40° = 34 N
The arrangement is symmetrical so that the equations for join C are the same, $T_{CD} = T_{AB}$ = 46 N

5



R = (65 N) cos 30° = 56 N T = 12 N + (65 N)cos 60° = 44.5 N = 45 N

Newton's laws

Pages 32–33

1 $F = ma$ $F = (740 \times 10^3\,\text{kg}) \times (0.05\,\text{m s}^{-2}) = 37\,000\,\text{N}$

2 $F = ma$ $12\,\text{N} = 30\,\text{kg} \times a$ $a = \dfrac{12\,\text{N}}{28\,\text{kg}} = 0.43\,\text{m s}^{-2}$

3 **a** For the aircraft: $(8.3 \times 10^4\,\text{N}) - T = (1.6 \times 10^4\,\text{kg})a$
For the glider: $T = (0.6 \times 10^4\,\text{kg})a$
$(8.3 \times 10^4\,\text{N}) - (0.6 \times 10^4\,\text{kg})a = (1.6 \times 10^4\,\text{kg})a$
$a = (8.3 \times 10^4\,\text{N}) \div [(1.6 + 0.6) \times 10^4\,\text{kg}] = 3.77\,\text{m s}^{-2} = 3.8\,\text{m s}^{-2}$ (2 s.f.)

b $T = (0.6 \times 10^4\,\text{kg}) \times (3.77\,\text{m s}^{-2}) = 2.3 \times 10^4\,\text{N}$

4 For the 5 kg mass: $T - (5 \times 9.81\,\text{N}) = (5\,\text{kg})a$
For the 8 kg mass: $(8 \times 9.81\,\text{N}) - T = (8\,\text{kg})a$ $a = \dfrac{(78.5\,\text{N}) - T}{8\,\text{kg}}$
$T - (49.1\,\text{N}) = \dfrac{(5\,\text{kg}) \times (78.5\,\text{N}) - (5\,\text{kg}) \times T}{8\,\text{kg}} = 49.1\,\text{N} - 0.63\,T$
$1.63T = 49.1\,\text{N} \times 2T = 60.2\,\text{N} = 60\,\text{N}$ (2 s.f.)
$a = 2.3\,\text{m s}^{-1}$

Work, energy and power

Pages 34–35

1 **a** $(22 \times 10^3\,\text{N}) \times (2.0 \times 10^3\,\text{m}) = 44\,\text{MJ}$
b $(620\,\text{N}) \times (150\,\text{m})\cos 80° = 16\,\text{kJ}$

2 $\Delta E_k = 16\,\text{kJ} = \frac{1}{2}mv^2 v = \sqrt{[2 \times (16.1 \times 10^3\,\text{J}) \times 9.81\,\text{m}\,\text{s}^{-2} \div (620\,\text{N})]} = 23\,\text{m}\,\text{s}^{-1}$
(This is over 50 mph, so clearly frictional forces are not small enough to be ignored!)

3 $\dfrac{(2500\,\text{N}) \times (15\,\text{m})}{(5.0\,\text{s})} = 7.5\,\text{kW} \quad \eta = \dfrac{(7.5\,\text{kW})}{(8.0\,\text{kW})} \times 100\% = 94\%$

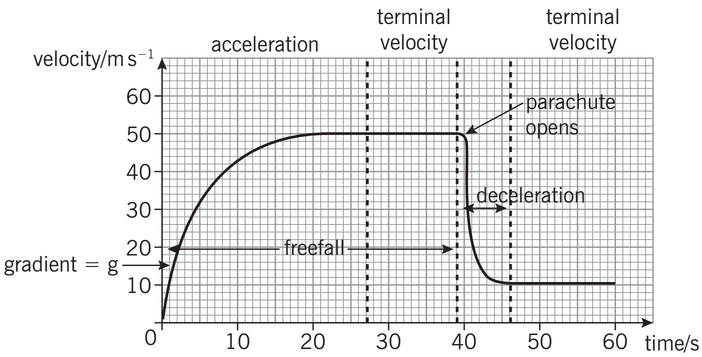
4 **a** $(28 \times 10^3\,\text{N}) \times (45\,\text{m}\,\text{s}^{-1}) = 1260 \times 10^3\,\text{W} = 1.3\,\text{MW}$
b $\dfrac{100}{30} \times 1.26\,\text{MW} = 4.2\,\text{MW}$

5 $\dfrac{92}{100} \times (55\,\text{W}) \times t = (15\,\text{m}) \times (9.81\,\text{m}\,\text{s}^{-2}) \times (2.5\text{m}) \quad t = \dfrac{368 \times 100\,\text{s}}{100\ 5060} = 7.3\,\text{s}$

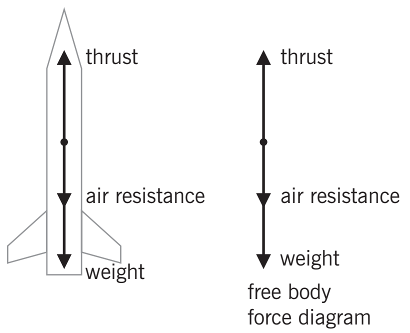
Vertical motion and gravity

Pages 36–37

1 At terminal velocity, air resistance = weight = $77\,\text{kg} \times 9.81\,\text{m}\,\text{s}^{-2} = 760\,\text{N}$
2 At terminal velocity, $W - D = 0 \quad mg = kv^2 \quad 65\,\text{kg} \times 9.81\,\text{m}\,\text{s}^{-2} = k(52\,\text{m}\,\text{s}^{-1})^2 \quad k = 0.29\,\text{kg}\,\text{m}^{-1}$
3 Because at such a great height there was less air so less air resistance. (He needed a spacesuit to breathe.)
4



5 **a**



The motion is upwards so drag is in the *same direction* as weight. Air resistance is 0.75 N and weight is $(0.3\,\text{kg}) \times (9.81\,\text{m}\,\text{s}^{-2})$ you don't need to work this out – but you can see it is much larger.
b At launch the speed = 0 so air resistance = 0 $F - W = ma \quad F - mg = ma$
$(5.0\,\text{N}) - (0.30\,\text{kg}) \times (9.81\,\text{m}\,\text{s}^{-2}) = (0.30\,\text{kg})a$
$(5.0\,\text{N}) - (2.94\,\text{N}) = (0.30\,\text{kg})a$
$a = \dfrac{(2.06\,\text{N})}{(0.30\,\text{kg})} = 6.9\,\text{m}\,\text{s}^{-2}$
c After 2.0 s the resultant upward force is $F - W - D = (4.0\,\text{N}) - (2.94\,\text{N}) - (0.75\,\text{N}) = (0.310\,\text{N})$
$(0.310\,\text{N}) = (0.30\,\text{kg})\ a\ a = 1.0\,\text{m}\,\text{s}^{-2}$

6 Take down as positive: $(68\,\text{kg}) \times (9.81\,\text{m}\,\text{s}^{-2}) - (720\,\text{N}) = (68\,\text{kg}) \times a$
$a = (9.81\,\text{m}\,\text{s}^{-2}) - \dfrac{(720\,\text{N})}{(68\,\text{kg})} = = (9.81\,\text{m}\,\text{s}^{-2}) - (10.6\,\text{m}\,\text{s}^{-2}) = -0.79\,\text{m}\,\text{s}^{-2}$
The acceleration is in an upward direction, so she is slowing down.

Density, pressure and upthrust

Pages 38–39

1 Density, $\rho = \dfrac{m}{V} = \dfrac{(1.31\,\text{kg})}{(5.0 \times 10^{-2}\,\text{m})^3} = 1.0 \times 10^4\,\text{kg}\,\text{m}^{-3}$
2 $m = \rho V = (8.9 \times 10^3\,\text{kg}\,\text{m}^{-3}) \times [\pi (0.25 \times 10^{-3}\,\text{m})^2 \times 150\,\text{m}] = 0.26\,\text{kg}$
3 Volume of mercury = $5.5\,\text{kg} - 0.45\,\text{kg} = 5.05\,\text{kg} \quad V = \dfrac{m}{\rho} = \dfrac{(5.05\,\text{kg})}{(13.6 \times 10^3\,\text{kg}\,\text{m}^{-3})} = 3.71 \times 10^{-4}\,\text{m}^3$

4 $p = \frac{F}{A} = \frac{2.0\,\text{N}}{0.10 \times 10^{-6}\,\text{m}^2} = 2.0 \times 10^7\,\text{Pa}$

5 Maximum pressure will be on smallest face $A = 6.0 \times 10^{-2}\,\text{m} \times 8.0 \times 10^{-2}\,\text{m} = 48.0 \times 10^{-4}\,\text{m}^{-2}$

$F = W = mg = \rho Vg = (2700\,\text{kg m}^{-3}) \times (6.0 \times 10^{-2}\,\text{m} \times 8.0 \times 10^{-2}\,\text{m} \times 12.0 \times 10^{-2}\,\text{m}) \times (9.81\,\text{m s}^{-2})$

$p = \frac{F}{A} = \frac{15.3\,\text{N}}{48.0} \times 10^{-4}\,\text{m}^2 = 3.2\,k\text{Pa}$

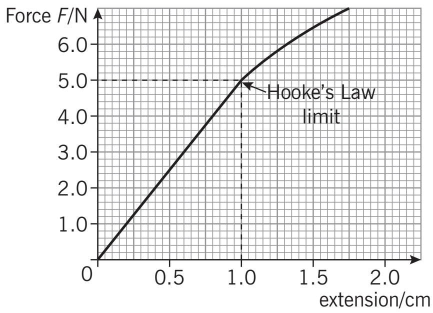
6 $T + U = mg \quad T = mg - U = (m_{\text{iron}} - m_{\text{liquid}})g = (m_{\text{iron}} - \rho_{\text{liquid}} V)g$

$V_{\text{liquid}} = V_{\text{iron}} = m_{\text{iron}} \div \rho_{\text{iron}} = (160 \times 10^{-3}\,\text{kg}) \div (8.0 \times 10^3\,\text{kgm}^{-3}) = 2.00 \times 10^{-5}\,\text{m}^3$

$T = [(160 \times 10^{-3}\,\text{kg}) - (810\,\text{kgm}^{-3})(2.00 \times 10^{-5}\,\text{m}^3)]9.81\,\text{m s}^{-2} = 1.4\,\text{N}$

Elasticity 1

Pages 40–41

1 a 

Note that extensions are found by subtracting the length at $F = 0$ (2.6 cm) from all the lengths.

b Force is directly proportional to extension from 0 to 5 N so graph goes through origin and is a straight line.
The Hooke's law limit is reached between 5 N and 6 N as the graph then curves. Force produces greater extension.

c $k = F/x = \frac{(5.0\,\text{N})}{(1.0 \times 10^{-2}\,\text{m})} = 500\,\text{N m}^{-1}$

2 $k = F/x = \frac{(160\,\text{N})}{(2.7 \times 10^{-2}\,\text{m})} = 5900\,\text{N m}^{-1}$

3 7 N

4 a $E = \frac{1}{2}(56\,\text{N}) \times (0.75 \times 10^{-3}\,\text{m}) = 0.021\,\text{J}$

b 0.021 J

$E = 517 \times (4\,\text{N}) \times (0.5 \times 10^{-3}\,\text{m}) = 1.0\,\text{J}$

Elasticity 2

Pages 42–43

1 a Young modulus $= \frac{23.0\ \text{G Pa}}{0.10} = 230\,\text{GPa}$ or $2.3 \times 10^{11}\,\text{Pa}$

b Young modulus $= \frac{16.0\,\text{GPa}}{0.180} = 89\,\text{GPa}$ or $8.9 \times 10^{10}\,\text{Pa}$

2 a 25.0 GPa **b** 22.0 GPa **c** 7.5 GPa

3 Brittle – stress $\propto$ strain until it snaps – no plastic deformation. Small extension for large stress.
In this example, large stress to break.
Strong – stress $\propto$ strain region, but small extension for large stress. Then very small plastic range, large stress to break.
Ductile – elastic then plastic, can be drawn out into a long wire before it breaks.
Plastic – no elastic range, deforms plastically

4 Strain, $\varepsilon = \frac{(2.5 \times 10^{-3}\,\text{m})}{(1.2\,\text{m})} = (2.08 \times 10^{-3})$

Area $A = \pi r^2 = \pi\,(0.9 \times 10^{-3}\,\text{m})^2 = 2.54 \times 10^{-6}\,\text{m}^2$

Force = Young modulus $\times\ \varepsilon A$

Force $= (2.0 \times 10^{11}\,\text{Pa})(2.08 \times 10^{-3})(2.54 \times 10^{-6}\,\text{m}^2) = 1100\,\text{N}$

5 Area $A = \pi r^2 = \pi\,(1.6 \times 10^{-3}\,\text{m})^2 = 8.04 \times 10^{-6}\,\text{m}^2$

Strain $= \frac{\text{stress}}{\text{Young modulus}}$

Extension $= \frac{Fl}{YA} = \frac{(30\,\text{N}) \times (2.0\,\text{m})}{\text{Young modulus} \times A\ (110 \times 10^9\,\text{Pa} \times (8.04 \times 10^{-6}\,\text{m}^2)} = 6.8 \times 10^{-5}\,\text{m}$

Stretch yourself

$F = (70\,\text{kg}) \times (9.81\,\text{N kg}^{-1}) = 687\,\text{N}$

Not safe to go beyond elastic limit, so maximum stress $= 5.0 \times 10^8\,\text{Pa}$

$\sigma = F/A$

So minimum area $= F/\sigma = 687\,\text{N} \div 5.0 \times 10^8\,\text{Pa} = 1.37 \times 10^{-6}\,\text{m}^2 = 1.4 \times 10^{-6}\,\text{m}^2$ (2 s.f.)

(Note as this is a safe minimum you should not round down your answer but round up and explain why you have done so.)

Resistance and resistivity

Pages 44–45

1 a $R = \dfrac{12\,\text{V}}{200 \times 10^{-3}\,\text{A}} = 60\,\Omega$ **b** $I = \dfrac{230\,\text{V}}{4.7 \times 10^{3}\,\Omega} = 49\,\text{mA}$ **c** $V = (1.8\text{A}) \times (5.0\,\Omega) = 9.0\,\text{V}$

2 a Add R values: **i** $1100\,\Omega$ **ii** $1110\,\text{k}\Omega$ or $1.11\,\text{M}\Omega$ ($1.1\,\text{M}\Omega$ to 2 s.f.) **iii** $3.5\,\Omega$

b i $\dfrac{1}{R} = \dfrac{3+2+1}{600\,\Omega} = \dfrac{1}{100\,\Omega}$ $R = 100\,\Omega$

ii $\dfrac{1}{R} = \dfrac{1 + 10 + 100}{1 \times 10^{6}\,\Omega} = \dfrac{111}{1 \times 10^{6}\,\Omega}$ $R = 9009\,\Omega = 9.0\,\text{k}\Omega$ (2 s.f.)

iii $\dfrac{1}{R} = 0.5 + 1 + 2$

$R = \dfrac{1}{3.5} = 0.29$

3 $A = \pi r^2 = \pi\,(0.20 \times 10^{-3}\,\text{m})^2 = 1.26 \times 10^{-7}\,\text{m}^2$ $\rho = \dfrac{(2.5\,\Omega) \times (1.26 \times 10^{-7}\,\text{m}^2)}{(1.8\,\text{m})} = 1.8 \times 10^{-7}\,\Omega\,\text{m}$

4 $A = \pi r^2 = \pi\,(0.25 \times 10^{-3}\,\text{m})^2 = 1.96 \times 10^{-7}\,\text{m}^2$ $R = \dfrac{(1.7 \times 10^{-8}\,\Omega\,\text{m}) \times (0.80\,\text{m})}{(1.96 \times 10^{-7}\,\text{m}^2)} = 0.069\,\Omega$

5 a $L = \dfrac{RA}{\rho} = \dfrac{(10.0\,\Omega) \times (2.0 \times 10^{-7}\,\text{m}^2)}{(1.1 \times 10^{-6}\,\Omega\,\text{m})} = 1.8\,\text{m}$

b $L = \dfrac{RA}{\rho} = \dfrac{(10.0\,\Omega) \times (2.0 \times 10^{-7}\,\text{m}^2)}{(5.6 \times 10^{-8}\,\Omega\,\text{m})} = 36\,\text{m}$ which is $1.8\,\text{m} \div 36\,\text{m} = 0.05$ times the length

OR $\dfrac{L(\text{nichrome})}{L(\text{tungsten})} = \dfrac{\rho(\text{tungsten})}{\rho\,(\text{nichrome})} = \dfrac{(5.6 \times 10^{-8}\,\Omega\,\text{m})}{(1.1 \times 10^{-6}\,\Omega\,\text{m})} = 0.05.$

So the nichrome is 0.05 times the length so the tungsten will be $1.8\,\text{m} \div 0.05 = 36\,\text{m}$.

Electric charge and current

Pages 46–47

1 $Q = 20\,\text{mA} \div 1.6 \times 10^{-19}\,\text{C} = 1.25 \times 10^{17}\,\text{s}^{-1}$. In one minute $N = 1.25 \times 10^{17}\,\text{s}^{-1} \times 60\,\text{s} = 7.5 \times 10^{18}$

2 $I = 6.0\,\text{mC} \div 2.0\,\text{s} = 3.0\,\text{mA}$

3 $v = I \div nAe = (2.0\,\text{A}) \div (8.0 \times 10^{28}\,\text{m}^{-3}) \times [\pi\,(0.30 \times 10^{-3}\,\text{m})^2] \times (1.60 \times 10^{-19}\,\text{C}) = 5.5 \times 10^{-4}\,\text{m}\,\text{s}^{-1}$

4 $I = (2.5 \times 10^{28}\,\text{m}^{-3}) \times [\pi\,(0.5 \times 10^{-3}\,\text{m})^2] \times (0.50 \times 10^{-3}\,\text{m}\,\text{s}^{-1}) \times (1.60 \times 10^{-19}\,\text{C}) = 1.6\,\text{A}$

5 $v = I \div nAe = (5.0\,\text{A}) \div (6.0 \times 10^{28}\,\text{m}^{-3}) \times (4.0 \times 10^{-6}\,\text{m}^2) \times (1.60 \times 10^{-19}\,\text{C}) = 1.3 \times 10^{-4}\,\text{m}\,\text{s}^{-1}$

6 Current through each R will be the same $\dfrac{V}{R}$ and through $2R$ will be a half of the others $\dfrac{V}{2R}$

$I_1 = I_3 = 2I_2$ $I_{\text{Total}} = 50\,\text{mA} = 2I_1 + 0.5$ $I_1 = 2.5\,I_1$ $I_1 = 20\,\text{mA}$ so $I_3 = 20\,\text{mA}$ and $I_2 = 10\,\text{mA}$

7 $I_1 = 12\,\text{V} \div 2\,\Omega = 6\,\text{A}$ $I_1 = I_2 + I_3$ $I_3 = 2I_2$ so $I_2 = 2\,\text{A}$ and $I_3 = 4\,\text{A}$ $R_2 = 6\,\text{V} \div 2\,\text{A} = 3\,\Omega$ $R_3 = 2\,R_2\,6\,\Omega$

Emf and potential difference

Pages 48–49

1 a $0.60\,\text{kW} = I \times 230\,\text{V}$ $I = 2.6\,\text{A}$

b $R = V \div I = 230\,\text{V} \div 2.6\,\text{A} = 88\,\Omega$

2 $E = Pt = IVt = 230\,\text{V} \times 0.40\,\text{A} \times 3600\,\text{s} = 3.3 \times 10^{5}\,\text{J}$

3 $9.0\,\text{V} \times 330\,\text{mA} = 2.97\,\text{W}$

4 $88\,\text{Ah} = 88 \times 3600\,\text{C} = 316\,800\,\text{C}$ $I = P \div V = 75\,\text{W} \div 12\,\text{V} = 6.25\,\text{A}$ $t = Q \div I = 316\,800 \div 6.25\,\text{A}$
$= 50\,688\,\text{s} = 14$ hours 5 minutes

5 $V_1 = 9\,\text{V}$ (same as $\mathcal{E}$) $V_2 = 5\,\text{V}$ ($9\,\text{V} - 4\,\text{V}$) $V_3 = 3\,\text{V}$ (same as across other $1\,\Omega$ resistor) $V_4 = 6\,\text{V}$ ($12\,\text{V} - 2 \times 3\,\text{V}$)

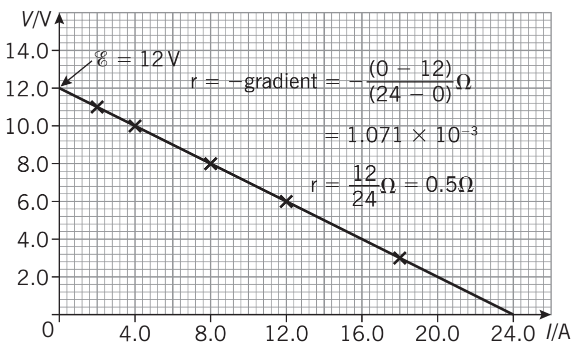
6 Total resistance $= 4.5\,\Omega$ current $= (9.0\,\text{V}) \div (4.5\,\Omega) = 2.0\,\text{A}$ pd $= IR = (2.0\,\text{A}) \times (4.0\,\Omega) = 8\,\text{V}$

7 a $(0.50\,\Omega)I + (6.0\,\text{V}) + (10.0\,\Omega)I = (12.0\,\text{V})$ gives $I = 0.57\,\text{A}$

b $V = (0.57\,\text{A})(10.0\,\Omega) = 5.7\,\text{V}$

c $12\,\text{V} - (6.0\,\text{V} + 5.7\,\text{V}) = 0.3\,\text{V}$

8



The potential divider and other circuits

Pages 50–51

1 a $V_{out} = \dfrac{(1.0\,k\Omega)}{(5.0\,k\Omega) + (1.0\,\Omega)} \times (9.0\,V) = 1.5\,V$

b $V_{out} = \dfrac{(330\,\Omega) \times (9.0\,V)}{(330\,\Omega) + (990\,\Omega)} = 2.3\,V$

c $\dfrac{(9.0 - 7.0\,V)}{(7.0\,V)} = \dfrac{R_1}{(680\,\Omega)}$

$R_1 = (680\,\Omega) \times 2.0 \div 7.0 = 190\,\Omega$

Stretch yourself

$\dfrac{(9.0 - 5.0\,V)}{(5.0\,V)} = \dfrac{R_1}{(450\,\Omega)}$

$R_1 = (450\,\Omega) \times 4.0 \div 5.0 = 360\,\Omega$

2 Pd across $3\,\Omega$ resistor = 12V. Pd across $8\,\Omega$ resistor = (60V − 12V) = 48V. Current through $8\,\Omega$ resistor = 48V ÷ $8\,\Omega$ = 6A
Current through R = 6A−4A = 2A. R = 12V ÷ 2A = $6\,\Omega$.

3 a 1st combination: $\dfrac{1}{R_T} = \dfrac{1}{1.0\,\Omega} + \dfrac{1}{3.0\,\Omega} + \dfrac{1}{6.0\,\Omega} = \dfrac{(6.0 + 2.0 + 1.0)}{6.0\Omega}$ $\quad R_T = (\tfrac{6}{9})\,\Omega = (\tfrac{2}{3})\,\Omega$

2nd combination: $\dfrac{1}{R_T} = 3 \times \left(\dfrac{1}{2.0\,\Omega}\right)$ $\quad R_T = (\tfrac{2}{3})\,\Omega$

Total $R_T = (\tfrac{2}{3})\,\Omega + (\tfrac{2}{3})\,\Omega = (\tfrac{4}{3})\,\Omega$

$V = IR \quad 1.5\,V = I_T\,(\tfrac{4}{3})\,\Omega$

$I_T = 1.5\,V \div (\tfrac{4}{3})\,\Omega = 1.125\,A = 1.1\,A$ (2 s.f.) or $(\tfrac{3}{2})\,V \div (\tfrac{4}{3})\,\Omega = (\tfrac{9}{8})\,A = 1.1\,A$ (2 s.f.)

b $I = \tfrac{1}{3}\,(\tfrac{9}{8})\,A = (\tfrac{3}{8})\,A = 0.38\,A$ (2 s.f.)

c $V = IR$

R_T for each parallel combination is the same, so the pd across each is (1.5 V) ÷ 2 = 0.75 V or $(\tfrac{3}{4})\,V$
[Alternatively use $2.0\,\Omega$ and $(\tfrac{3}{8})\,A$ to calculate V across the other combination and subtract from 1.5 V]
$I = (\tfrac{3}{4})\,V \div 3.0\,\Omega = (\tfrac{1}{4})\,A = 0.25\,A$

The photoelectric effect

Pages 52–53

1 $5.7 \times 10^{-19}\,J \div 1.60 \times 10^{-19}\,J$ per eV = 3.6 eV

2 a $E = (6.63 \times 10^{-34}\,J\,s) \times (3.0 \times 10^{8}\,m\,s^{-1}) \div (590 \times 10^{-9}\,m) = 3.37 \times 10^{-19}\,J = 3.4 \times 10^{-19}\,J$

b $3.37 \times 10^{-19}\,J \div 1.60 \times 10^{-19}\,J$ per eV = 2.1 eV

3 a $\phi = hf_0 \quad f_0 = 2.46\,eV\,(1.60 \times 10^{-19}\,J$ per eV$) \div 6.63 \times 10^{-34}\,J\,s = 5.94 \times 10^{14}\,Hz$

b $\tfrac{1}{2}m_e v_{max}^2 = (6.63 \times 10^{-34}\,J\,s)\,(3.0 \times 10^{8}\,m\,s^{-1}) \div (450 \times 10^{-9}\,m) - 2.46\,eV\,(1.60 \times 10^{-19}\,J$ per eV$)$

$4.42 \times 10^{-19}\,J - 3.93 \times 10^{-19}\,J = 0.49 \times 10^{-19}\,J = 4.9 \times 10^{-20}\,J$ or 0.31 eV

4 a $hf = \phi + \tfrac{1}{2}m_e v_{max}^2 \quad \phi = \dfrac{(6.63 \times 10^{-34}\,J\,s) \times (3.0 \times 10^{8}\,m\,s^{-1})}{(290 \times 10^{-9}\,m) \times (1.60 \times 10^{-19}\,J \text{ per eV})} + 2.19\,eV$

$\phi = 4.29\,eV - 2.19\,eV = 2.1\,eV$

b $\tfrac{1}{2}\,m_e v_{max}^2 = \dfrac{(6.63 \times 10^{-34}\,J\,s) \times (3.0 \times 10^{8}\,m\,s^{-1})}{(350 \times 10^{-9}\,m) \times (1.60 \times 10^{-19}\,J \text{ per eV})} - 2.1\,eV = 1.5\,eV$

Momentum

Pages 54–55

1 $mv = 60\,000\,kg\,m\,s^{-1}$

2 Momentum = $2.0\,kg \times 3.0\,m\,s^{-1}$ per second
Change in momentum = $-6.0\,kg\,m\,s^{-1}$ per second $F = -6.0\,N$

3 a $F\Delta t = \Delta mv = 0.50\,kg\,(12\,m\,s^{-1} - 6.0\,m\,s^{-1}) = 3.0\,kg\,m\,s^{-1}$ in original direction

b $F\Delta t = \Delta mv = 0.50\,kg(-12\,m\,s^{-1} - 6.0\,m\,s^{-1}) = -9.0\,kg\,m\,s^{-1}$ (i.e. in opposite direction)

4 By Cons. of mom^m (conservation of momentum): before: $(0.80\,kg)(1.5\,m\,s^{-1}) - (1.2\,kg)(1.8\,m\,s^{-1}) = -0.96\,kg\,m\,s^{-1}$
After: $[(0.80+1.2)\,kg] \times v = 2.0\,kg \times v$
$(2.0\,kg)v = -0.96\,kg\,m\,s^{-1} \quad v = -0.48\,kg\,m\,s^{-1}$

5 By Cons. of mom^m: $(3600\,kg)v = (1100\,kg)(6.0\,m\,s^{-1}) + (2500\,kg)(5.0\,m\,s^{-1}) = 6600\,kg\,m\,s^{-1} + 12500\,kg\,m\,s^{-1}$
$v = 19\,100 \div 3600 = 5.3\,m\,s^{-1}$

Momentum and energy

Pages 56–57

1 a $m\,(0.3m\,s^{-1}) + m(-0.2m\,s^{-1}) = m(-0.2m\,s^{-1}) + mv_2$

$v_2 = +0.3\,m\,s^{-1}$

b Use $\tfrac{1}{2}mv^2$ to show total KE before = total KE after. Therefore collision is elastic.

2 a $(5.0\,kg)(20.0\,m\,s^{-1}) + (0) = (5.0\,kg)(-6.67\,m\,s^{-1}) + (10.0\,kg)v_2$

$v_2 = 13.35\,m\,s^{-1} = 13\,m\,s^{-1}$ (2 s.f.)

b KE before $= \frac{1}{2}(5.0\,kg)(20.0\,m\,s^{-1})^2 = 1000\,J$

KE after $= \frac{1}{2}(5.0\,kg)(6.67\,m\,s^{-1})^2 + \frac{1}{2}(10.0\,kg)(13.35\,m\,s^{-1})^2 = 1002\,J = 1000\,J$ (3 s.f.)

Therefore is elastic to 3 s.f.

3 a $(4u)(1.0 \times 10^6\,m\,s^{-1}) + (0) = (4u)(0.60 \times 10^6\,m\,s^{-1}) + (1u)v_2$

$v_2 = 1.6 \times 10^6\,m\,s^{-1}$ to the right

b KE before $= \frac{1}{2}(4u)(1.0 \times 10^6\,m\,s^{-1})^2$

KE after $= \frac{1}{2}(4u)(0.60 \times 10^6\,m\,s^{-1})^2 + \frac{1}{2}(1u)(1.6 \times 10^6\,m\,s^{-1})^2 = \frac{1}{2}(4u)(1.0 \times 10^6\,m\,s^{-2})$

Therefore is elastic

4 $(7.0 \times 10^{-3}\,kg)(210\,m\,s^{-1}) = (1.507\,kg)v$

$v = 0.975\,m\,s^{-1}$

$\Delta\frac{1}{2}mv^2 = \Delta mgh$ $h = v^2/2g = (0.975\,m\,s^{-1})^2 \div [\,2 \times (9.81\,m\,s^{-2})] = 0.484\,m = 4.8\,cm$ (2 s.f.)

5 a By Cons. of mom^m: $(1.0\,kg)(5.0\,m\,s^{-1}) = (7.0\,kg)\ v_1$

$v_1 = (5.0/7.0)\,m\,s^{-1}$

$(7.0\,kg)(5.0/7.0)\,m\,s^{-1} = (10.0\,kg)\ v_2$

$v_2 = (5.0/10.0)\,m\,s^{-1} = 0.5\,m\,s^{-1}$

b Initial KE $= \frac{1}{2} \times (1.0\,kg)(5.0\,m\,s^{-1})^2 = 12.5\,J$

Final KE $= \frac{1}{2} \times (10.0\,kg)(0.5\,m\,s^{-1})^2 = 1.25\,J$

KE lost $= 12.5\,J - 1.25\,J = 11.25\,J$

6 Mom^m before: $(42\,kg)u + 0$

Mom^m after: $(42\,kg + 12\,kg)v$

By Cons. of mom^m: $(42\,kg)u = (54\,kg)v$ equation 1

For swing: loss of E_k = gain in E_p

$\frac{1}{2}(54\,kg)v^2 = (54\,kg)(9.81\,m\,s^{-2})(1.6\,m)$

$v^2 = 2(9.81\,m\,s^{-2})(1.6\,m) = 31.4\,m^2\,s^{-2}$

$v = 5.60\,m\,s^{-1}$

substitute in equation 1 $u = (54\,kg)(5.60\,m\,s^{-1}) \div (42\,kg) = 7.2\,m\,s^{-1}$

Answers to A2 spreads

Circular motion

Pages 58–59

1 a $\pi/3$ radians **b** π radians **c** 3π radians

2 a 45° **b** 270° **c** 30°

d $(1 \times 360°) \div (2\pi) = 57.3° = 57°$ (2 s.f.) **e** $(12.6 \times 360°) \div (2\pi) = 720°$ (2 s.f.)

3 $\omega = 2\,\pi/(0.20\,s) = 31$ radians s^{-1}

4 a $\omega = v/r = (12\,m\,s^{-1}) \div (6.0\,m) = 2.0$ radians s^{-1}

b $F = mv^2/r = (2.0\,kg)(12\,m\,s^{-1})^2/(6.0\,m) = 48\,N$

Stretch yourself

At the top of the circular path: $mg + T_{min} = mv^2/r$

$T_{min} = m(v^2/r - g) = (6.0\,kg)[(5.0\,m\,s^{-1})^2/(1.5\,m) - 9.81\,m\,s^{-2}] = 41\,N$ (2 s.f.)

At the bottom of the circular path: $T_{max} - mg = mv^2/r$

$T_{max} = m(v^2/r + g) = (6.0\,kg)[(5.0\,m\,s^{-1})^2/(1.5\,m) + 9.81\,m\,s^{-2}] = 160\,N$ (2 s.f.)

SHM 1

Pages 60–61

1 a $f = v_{max}/2\pi A$ $f = (0.24\,m\,s^{-1}) \div 2\pi(48 \times 10^{-3}\,m) = 0.795\,Hz$ $T = 1/f = 1.3\,s$

b $a_{max} = -\omega^2 A = -[2\pi(0.795Hz)]^2(48 \times 10^{-3}\,m) = -\ 1.2\,m\,s^{-2}$

2 $v_{max} = \pm 2\pi f A$ $(4.2\,m\,s^{-1}) = \pm 2\pi(512\,Hz)A$

$A = (4.2\,m\,s^{-1}) \div 2\pi(512\,Hz) = 1.3 \times 10^{-3}\,m$

3 $x = (2.0\,mm)\cos(3\pi t)$

a $2\pi f = 3\pi$ $f = 1.5\,Hz$

b $T = 1/(1.5\,Hz) = 0.67\,s$

c $A = 2.0\,mm$

d $v_{max} = \pm 2\pi f A = \pm 2\pi(1.5\,Hz)(2.0 \times 10^{-3}\,m) = 1.9 \times 10^{-2}\,m\,s^{-1}$

e $a_{max} = -(2\pi f)^2 A = -[2\pi(1.5\,Hz)]^2(2.0 \times 10^{-3}\,m) = -\ 0.18\,m\,s^{-2}$

f $t = 0.060s$ $x = (2.0 \times 10^{-3}\,m)\cos[3\pi(0.060s)] = 1.69 \times 10^{-3}\,m = 1.7\,mm$ (2 s.f.)

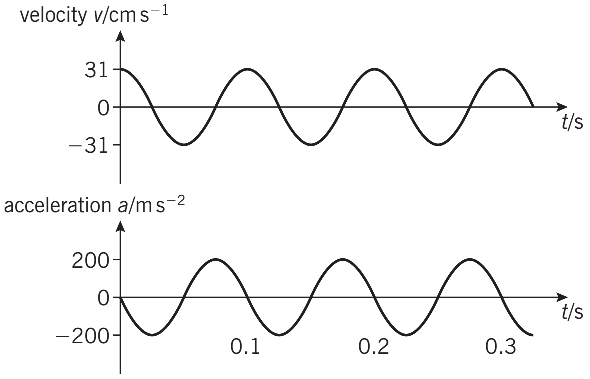
g $v = \pm 2\pi(1.5\,Hz)\sqrt{[(2.0 \times 10^{-3}\,m)^2 - (1.69 \times 10^{-3}\,m)^2]} = \pm 0.010\,m\,s^{-1}$

h $a = -[2\pi(1.5\,Hz)]^2(1.69 \times 10^{-3}\,m) = -0.15\,m\,s^{-2}$

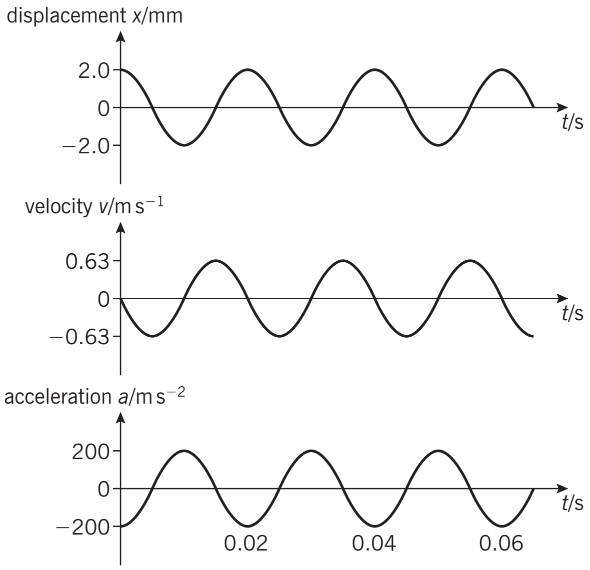
Stretch yourself
80 revs s^{-1} $\omega = 80 \times 2\pi$ radians s^{-1} = 160π radians s^{-1}
$a_{max} = -\omega^2 A = -(160\pi \text{ radians s}^{-1})^2(6.0 \times 10^{-2}\text{ m}) = 15\,200\text{ m s}$
$F_{max} = (0.55\text{ kg})(15\,200\text{ m s}^{-2}) = 8300\text{ N}$

SHM 2

Pages 62–63

1 

$\omega = 2\pi\,(10\text{ Hz}) = 63\text{ radians s}^{-1}$
$v = (63\text{ radians s}^{-1})(5.0\text{ cm}) = 31\text{ cm s}^{-1}$
$a = (63\text{ radians s}^{-1})^2(5.0\text{ cm}) = 19\,800\text{ cm s}^{-2} = 2.0 \times 10^2\text{ m s}^{-2}$

2 

$\omega = 2\pi\,(50\text{ Hz}) = 100\text{ radians s}^{-1}$ $A = 2\text{ mm}$ $T = 1/(50\text{ Hz}) = 0.02\text{ s}$
$v = \pm\,100\pi(2\text{ mm}) = 0.63\text{ m s}^{-1} = 0.6\text{ m s}^{-1}$(1 s.f.)
$a = -(100\pi)^2(2\text{ mm}) = 197\text{ m s}^{-2} = 2 \times 10^2\text{ m s}^{-2}$ (1 s.f.)

3 $E_K = \frac{1}{2}m(2\pi f A)^2 = \frac{1}{2} \times (0.250\text{ kg})[2\pi(5.5\text{ Hz})(7.5 \times 10^{-2}\text{ m})]^2 = 0.83\text{ J}$
As graph on Pages 63 with $A = 7.5$ cm and $E_{total} = 0.83$ J

4 **a** $E_P = \frac{1}{2}kx^2 = \frac{1}{2}(1.2 \times 10^2\text{ N m}^{-1}) \times (0.15\text{ m})^2 = 1.4\text{ J}$
b as graph on Pages 63 with $A = 0.15$ m and $E_{total} = 1.4$ J
c as graph on Pages 63 with $A = 0.15$ m and $E_{total} = 1.4$ J for t up to $T = 0.2$ s

Examples of SHM

Pages 64–65

1 $2\pi f^2 = g/l\; 2\pi f = \sqrt{(g/l)}$ $f = (1/2\pi)\sqrt{(g/l)}$ $T = 1/f = 2\pi\sqrt{(l/g)}$
2 $1.0\text{s} = 2\pi\sqrt{(l/9.81\text{ m s}^{-2})}$ $l = 0.25$ m
3 $T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{[(0.64\text{ kg})/(48\text{ N m}^{-1})]} = 0.73\text{ s}$
4 **a** $T = 1/f = 1/(2.0\text{ Hz}) = 0.5\text{ s}$ **b** $0.5\text{s} = 2\pi\sqrt{[(0.40\text{ kg})/k]} = 63\text{ N m}^{-1}$

Temperature scales and the gas laws

Pages 66–67

1 $\theta(°\text{C}) = 77\text{ K} - 273.15\text{ K} = -196.15\,°\text{C}$
2 $T(\text{K}) = -78\,°\text{C} + 273.15\text{ K} = 195.15\text{ K}$

3 $p_1 = 3.1 \times 10^5\,\text{Pa}^{-2}$ $p_2 = 1.0 \times 10^5\,\text{Pa}^{-2}$ $V_1 = 4.2 \times 10^{-6}\,\text{m}^3$ $V_2 = ?$

$$V_2 = \frac{p_1 V_1}{p_2} = \frac{(3.1 \times 10^5\,\text{Pa}^{-2})(4.2 \times 10^{-6}\,\text{m}^3)}{(1.0 \times 10^5\,\text{Pa}^{-2})} = 1.3 \times 10^{-5}\,\text{m}^3$$

4 $p_1 = 6.0 \times 10^5\,\text{Pa}^{-2}$ $T_1 = (273.15+30)\text{K}$ $T_2 = (273.15+21)\text{K}$ $p_2 = ?$

$$p_2 = \frac{p_1 T_2}{T_1} = \frac{(6.0 \times 10^5\,\text{Pa}^{-2})(294.15\,\text{K})}{(303.15\,\text{K})} = 5.8 \times 10^5\,\text{Pa}^{-2}$$

5 $V_1 = 0.011\,\text{m}^3$ $T_1 = (273.15 + 21)\text{K}$ $T_2 = (273.15 + 13)\text{K}$ $V_2 = ?$

$$V_2 = \frac{V_1 T_2}{T_1} = \frac{(0.011\,\text{m}^3)(286.15\,\text{K})}{(294.15\,\text{K})} = 0.011\,\text{m}^3$$ so the change is negligible.

An ideal gas

Pages 68–69

1 $V_1 = 7200\,\text{m}^3$ $T_1 = (273+22)\text{K}$ $p_1 = 1.0 \times 10^5\,\text{Pa}$ $V_2 = 25\,\text{m}^3$ $T_2 = (273+12)\text{K}$ $p_2 = ?$

$$p_2 = \frac{p_1 V_1 T_2}{T_1 V_2} = \frac{(1.0 \times 10^5\,\text{Pa})(7200\,\text{m}^3)(285\,\text{K})}{(295\,\text{K})(25\,\text{m}^3)} = 2.8 \times 10^7\,\text{Pa}$$

2 $V_2 = \frac{V_1}{2}$ $p_2 = 3p_1$ $\frac{T_2}{T_1} = ? = \frac{p_2 V_2}{p_1 V_1} = \frac{3p_1 V_1/2}{p_1 V_1} = \frac{3}{2} = 1.5$

Stretch yourself

$T_1 = (273+15)\text{K}$ $p_1 = 1.01 \times 10^6\,\text{Pa}$ $V_2 = 2V_1$ $T_2 = (273+55)\text{K}$

$$p_2 = \frac{p_1 V_1 T_2}{T_1 V_2} = \frac{(1.01 \times 10^6\,\text{Pa})(V_1)(328\,\text{K})}{(288\,\text{K})(2V_1)} = 5.8 \times 10^5\,\text{Pa}$$

3 $V = 8.0 \times 10^{-3}\,\text{m}^3$ $T = (273+25)\text{K}$ $p = 9.0 \times 10^5\,\text{Pa}$

a $$n = \frac{pV}{RT} = \frac{(9.0 \times 10^5\,\text{Pa})(8.0 \times 10^{-3}\,\text{m}^3)}{(8.31\,\text{JK}^{-1}\,\text{mol}^{-1})(298\,\text{K})} = 2.9\,\text{mol}$$

b number $= nN_A = (2.9\,\text{mol})(6.02 \times 10^{23}\,\text{mol}^{-1}) = 1.8 \times 10^{24}$

4 $V = ?$ $n = 1\,\text{mol}$ $T = 273\,\text{K}$ $p = 1.01 \times 10^5\,\text{Pa}$

$$V = \frac{nRT}{p} = \frac{(1\,\text{mol})(8.31\,\text{J}\,\text{K}^{-1}\,\text{mol}^{-1})(273\text{K})}{(1.01 \times 10^5\,\text{Pa})} = 0.0224\,\text{m}^3 = 22.4\,\text{l}$$

In a 'show that' question give one more figure than in the question to show that you have calculated and not copied it.

5 $V = 27\,\text{m}^3$ $T = (273+22)\text{K}$ $p = 1.01 \times 10^5\,\text{Pa}$ $m = ?$ $M = 0.029\,\text{kg}\,\text{mol}^{-1}$

$$pV = \frac{m}{M}RT \quad m = \frac{pVM}{RT} = \frac{(1.01 \times 10^5\text{Pa})(27\,\text{m}^3)(0.029\,\text{kg}\,\text{mol}^{-1})}{(8.31\,\text{JK}^{-1}\,\text{mol}^{-1})(295\,\text{K})} = 32\,\text{kg}$$

Stretch yourself

$\rho = ? = m/V$ $T = (273+25)\text{K}$ $p = 1.01 \times 10^5\,\text{Pa}$ $M = 0.002\,\text{kg}\,\text{mol}^{-1}$

$$pV = \frac{m}{M}RT$$

$$\rho = \frac{m}{V} = \frac{pM}{RT} = \frac{(1.01 \times 10^5\text{Pa})(0.002\,\text{kg}\,\text{mol}^{-1})}{(8.31\,\text{J}\,\text{K}^{-1}\,\text{mol}^{-1})(298\,\text{K})} = 0.082\,\text{kg}\,\text{m}^{-3}$$

Kinetic theory

Pages 70–71

1 a $[6 \times 10.0 + 9 \times 20.0 + 10 \times 30.0 + 12 \times 40.0 + 13 \times 50.0 + 10 \times 60.0 + 7 \times 70.0 + 5 \times 80.0] \div \text{total } N$

Where total $N = [6 + 9 + 10 + 12 + 13 + 10 + 7 + 5] = 72$

Mean $= 3160 \div 72 = 44\,\text{m}\,\text{s}^{-1}$ (2 s.f.)

b $c_{rms} = \sqrt{[(6 \times 10.0^2 + 9 \times 20.0^2 + 10 \times 30.0^2 + 12 \times 40.0^2 + 13 \times 50.0^2 + 10 \times 60.0^2 + 7 \times 70.0^2 + 5 \times 80.0^2) \div 72]}$

$c_{rms} = \sqrt{[(600 + 3600 + 9000 + 19\,200 + 32\,500 + 36\,000 + 34\,300 + 32\,000) \div 72]}$

$c_{rms} = \sqrt{[167\,200 \div 72]} = 48\,\text{m}\,\text{s}^{-1}$ (2 s.f.)

2 $p = 1.01 \times 10^5\,\text{Pa}$ $\rho = 1.5\,\text{kg}\,\text{m}^{-3}$

$$c_{rms}{}^2 = \frac{3p}{\rho} = \frac{3(1.01 \times 10^5\,\text{Pa})}{(1.5\,\text{kg}\,\text{m}^{-3})} = 2.0 \times 10^5\,\text{m}^2\text{s}^{-2} \quad c_{rms} = 450\,\text{m}\,\text{s}^{-1}$$

3 a i $T = (273+0)\,\text{K}$

$$E_K = \frac{kTn}{2N_A} = \frac{3}{2}(1.38 \times 10^{-23}\,\text{J}\,\text{K}^{-1})(273\,\text{K})(1\,\text{mol})(6.02 \times 10^{23}\,\text{mol}^{-1}) = 3.4 \times 10^3\,\text{J}$$

ii $T = (273+100)\,\text{K}$ $E_K = (3.4 \times 10^3\,\frac{\text{J})(373\,\text{K})}{(273\,\text{K})} = 4.6 \times 10^3\,\text{J}$

b i $$\frac{(3.4 \times 10^3\,\text{J})}{(1\,\text{mol})(6.02 \times 10^{23}\,\text{mol}^{-1})} = 5.6 \times 10^{-21}\,\text{J}$$

ii $$\frac{(4.6 \times 10^3\,\text{J})}{(1\,\text{mol})(6.02 \times 10^{23}\,\text{mol}^{-1})} = 7.7 \times 10^{-21}\,\text{J}$$

4 $c_{\text{rms}} = 515\,\text{m}\,\text{s}^{-1}$ $\quad T = ?\quad \frac{1}{2}mc_{\text{rms}}^2 = (3/2)\,kT$

$$T = \frac{mc_{\text{rms}}^2}{3k} = \frac{(515\,\text{m}\,\text{s}^{-1})^2(0.028\,\text{kg}\,\text{mol}^{-1})}{3(1.38\times10^{-23}\,\text{J}\,\text{K}^{-1})(6.02\times10^{23}\,\text{mol}^{-1})} = 298\,\text{K (or 25 °C)}$$

5 a $p_1 = 2.7\times10^3\,\text{Pa}\quad T_1 = 47\text{K}\quad T_2 = ?\quad p_2 = 2.4\times10^4\,\text{Pa}$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}\quad T_2 = \frac{(2.4\times10^4\,\text{Pa})(T_1 = 47\,\text{K})}{(2.7\times10^3\,\text{Pa})} = 418\,\text{K} = 420\,\text{K (2 s.f.)}$$

b $c_{\text{rms1}} = 8.0\times10^2\,\text{m}\,\text{s}^{-1}\ T_1 = 47\,\text{K}\quad c_{\text{rms2}} = ?\quad T_2 = 418\,\text{K}$

$$\frac{1}{2}mc_{\text{rms}}^2 = (3/2)\,kT\ mc_{\text{rms}}^2 = 3kT\,\frac{c_{\text{rms1}}^2}{c_{\text{rms2}}^2} = \frac{T_1}{T_2}$$

$$c_{\text{rms2}}^2 = \frac{(8.0\times10^2\,\text{m}\,\text{s}^{-1})^2\,(418\,\text{K})}{(47\,\text{K})} = 5.692\times10^6\,\text{m}^2\,\text{s}^{-2}$$

$c_{\text{rms2}} = 2400\,\text{m}\,\text{s}^{-1}$

Gravity

Pages 72–73

1 $$F = \frac{GmM}{r^2} = \frac{(6.67\times10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2})(1.99\times10^{30}\,\text{kg})(5.98\times10^{24}\,\text{kg})}{(1.50\times10^{11}\,\text{m})^2} = 3.5\times10^{22}\,\text{N}$$

2 $$F = \frac{6.67\times10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}\times10.0\,\text{kg}\times\ 10.0\,\text{kg}}{10\,\text{m}\times10.0\,\text{m}} = 6.67\times10^{-11}\,\text{N}$$

3 $$g = -\frac{GM}{r^2} = -\frac{(6.67\times10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2})(1.90\times10^{27}\,\text{kg})}{(7.14\times10^7\,\text{m})^{22}} = -\ 24.8\,\text{N}\,\text{kg}^{-1}$$

4 Moon: $mg = -\dfrac{GmM}{r^2}$ Moon: $g = \dfrac{g_{\text{Earth}}}{6}\quad r = 1.74\times10^6\,\text{m}\,\text{s}^{-1}$

$$\frac{(9.81\,\text{m}\,\text{s}^{-2})}{6} = \frac{(6.67\times10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2})M}{(1.74\times10^6\,\text{m}\,\text{s}^{-1})^2}$$

$M = 7.4\times10^{22}\,\text{kg}$

Stretch yourself

a $g = -\dfrac{GM}{r^2}\quad r = (1000\,\text{m} + 6.38\times10^6\,\text{m})\ M = 5.98\times10^{24}\,\text{kg}$

$$g = -\frac{(6.67\times10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2})(\ 5.98\times10^{24}\,\text{kg}\ )}{[(6380\ +\ 1)\times10^3\,\text{m}]^2} = 9.80\,\text{m}\,\text{s}^{-2}$$

b When you repeat this calculation with $r = 6380\times10^3\,\text{m}$ the answer is still $9.80\,\text{m}\,\text{s}^{-2}$, so up to 1000 m yes it is appropriate because, (to 2 s.f.) g doesn't change whether you use $6380\times10^3\,\text{m}$ or $6381\times10^3\,\text{m}$.

Orbital motion

Pages 74–75

1 $T = 24\times3600\text{s}\quad \omega = 2\pi/(24\times60\times60\text{s})\ M = 5.98\times10^{24}\,\text{kg}$

$$\frac{GmM}{r^2} = mr\omega^2\quad r^3 = \frac{GM}{\omega^2} = \frac{(6.67\times10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2})(5.98\times10^{24}\,\text{kg})(24\times60\times60\,\text{s})^2}{4\pi^2}$$

$r^3 = 7.542\times10^{22}\,\text{m}^3$

$r = 4.2\times10^7\,\text{m}$

2 Graph similar to worked example, giving mass of the Sun $\sim 2\times10^{30}\,\text{kg}$.

Electric fields

Pages 76–77

1 $$F = \frac{(3.0\times10^{-9}\,\text{C})(6.0\times10^{-9}\,\text{C})}{4\pi(8.85\times10^{-12}\,\text{C}^2\,\text{N}^{-1}\,\text{m}^{-2})(0.3\,\text{m})^2} = 1.8\times10^{-6}\,\text{N}$$

2 $$E = -\frac{(1.60\times10^{-19}\,\text{C})}{4\pi(8.85\times10^{-12}\,\text{C}^2\,\text{N}^{-1}\,\text{m}^{-2})(10\times10^{-2}\,\text{m})^2} = -1.4\times10^{-7}\ \text{(i.e. towards the electron)}$$

3 $$E = F/q\ E = \frac{(4.5\times10^{-3}\,\text{N})}{(6.0\times10^{-9}\,\text{C})} = 7.5\times10^5\,\text{N}\,\text{C}^{-1}$$

4 a $$E = V/d = \frac{(3.5\times10^3\,\text{V})}{(1.8\times10^{-2}\,\text{m})} = 1.9\times10^5\,\text{V}\,\text{m}^{-1}$$

b $F = Eq = (1.9\times10^5\,\text{V}\,\text{m}^{-1})(-1.60\times10^{-19}\,\text{C}) = 3.0\times10^{-14}\,\text{N}$ towards the positive plate.

5 $\text{V}\,\text{m}^{-1} = \text{J}\,\text{C}^{-1}\,\text{m}^{-1} = \text{N}\,\text{m}\,\text{C}^{-1}\,\text{m}^{-1} = \text{N}\,\text{C}^{-1}$

Magnetic fields

Pages 78–79

1 $F = (2.0 \times 10^{-3}\text{T})(360 \times 10^{-3}\text{A})(12 \times 10^{-2}\,\text{m}) = 8.6 \times 10^{-5}\,\text{N}$

2 a $v = rBQ/m = \dfrac{(14 \times 10^{-2}\,\text{m})(0.35\text{T})(1.60 \times 10^{-19}\,\text{C})}{(1.67 \times 10^{-27}\,\text{kg})} = 4.69 \times 10^{6}\,\text{m s}^{-1} = 4.7 \times 10^{6}\,\text{m s}^{-1}$ (2 s.f.)

b $r = \dfrac{(9.11 \times 10^{-31}\,\text{kg})(4.69 \times 10^{6}\,\text{m s}^{-1})}{(0.35\text{T})(1.60 \times 10^{-19}\,\text{C})} = 7.6 \times 10^{-5}\,\text{m}$ (or $r_e = m_e v/BQ \quad r_p = m_p v/BQ \quad r_e = r_p \times m_e/m_p$)

3 $v = \sqrt{(2\,E_K/m)} = \sqrt{(2 \times 2.5 \times 10^{-14}\,\text{J}) \div (1.67 \times 10^{-27}\,\text{kg})} = 5.47 \times 10^{6}\,\text{ms}^{-1}$

$r = \dfrac{mv}{BQ} = \dfrac{(1.67 \times 10^{-27}\,\text{kg})(5.47 \times 10^{6}\,\text{m s}^{-1})}{(0.27\,\text{T})(1.6 \times 10^{-19}\,\text{C})} = 0.21\,\text{m}$ (2 s.f.)

Electromagnetic induction

Pages 80–81

1 $A = \pi r^2 = \pi(15 \times 10^{-2}\,\text{m})^2 = 0.071\,\text{m}^2$ $\phi = BA = (0.25\,\text{T})(0.071\,\text{m}^2) = 0.018\,\text{Wb}$

2 $A = 100\,\text{cm}^2$ $N = 50$ turns $\theta = 60°$ $B = 7.5 \times 10^{-3}\,\text{T}$

$N\phi = (50\text{ turns})(100 \times 10^{-4}\,\text{m}^2)(7.5 \times 10^{-3}\,\text{T})\cos 60° = 1.9 \times 10^{-3}\,\text{Wb turns}$

3 $l = 42\,\text{m}$ $v = [(900 \times 10^{3} \div 3600)]\,\text{m s}^{-1} = 250\,\text{m s}^{-1}$ $B = 8.77 \times 10^{-6}\,\text{T}$ $\theta = 64°$

$E = (8.77 \times 10^{-6}\,\text{T})(42\,\text{m})(250\,\text{m s}^{-1})\cos 64° = 0.040\,\text{V}$

4 $N = 50$ turns $A = 45 \times 10^{-4}\,\text{m}^2$ $B = 1.2 \times 10^{-2}\,\text{T}$ $\theta = 0°$

$E = -\dfrac{\Delta(NBA\cos\theta)}{\Delta t}$ $(NBA\cos\theta) = (50\text{ turns})(1.2 \times 10^{-2}\,\text{T})(45 \times 10^{-4}\,\text{m}^2)(1) = 2.7 \times 10^{-3}\,\text{Wb turns}$

a $E = \dfrac{-(0-2.7 \times 10^{-3}\,\text{Wb turns})}{0.050\,\text{s}} = 0.054\,\text{V}$

b $E = \dfrac{-(50\text{ turns})[1.6 - 1.2 \times 10^{-2}\,\text{T}](45 \times 10^{-4}\,\text{m}^2)(1)}{(0.10\,\text{s})} = -\,9.0 \times 10^{-3}\,\text{V}$ (i.e. in opposite direction)

c $E = \dfrac{-(50\text{ turns})[-1.2 - 1.2 \times 10^{-2}\,\text{T}](45 \times 10^{-4}\,\text{m}^2)(1)}{(0.20\,\text{s})} = 0.027\,\text{V}$ (in same direction as a)

Capacitors in circuits

Pages 82–83

1 $Q = CV = (450 \times 10^{-6}\,\text{F})(2.2 \times 10^{3}\,\text{V}) = 0.99\,\text{C}$

2 a $C = C_1 + C_2 = 18\,\mu\text{F}$

b $C = \dfrac{(6.0 \times 18)}{(6.0 + 18)}\,\mu\text{F} = 4.5\,\mu\text{F}$

c i Charge is the same as the total charge Q $Q = (4.5\,\mu\text{F})(12\,\text{V}) = 5.4 \times 10^{-5}\,\text{C}$

ii The total charge on the 2 capacitors is 5.4×10^{-5} C. The two capacitors are identical so the charge on each is half the total charge: $Q = 0.5 \times (5.4 \times 10^{-5}\,\text{C}) = 2.7 \times 10^{-5}\,\text{C}$

d $V = ?$ $\quad Q = CV = \mu\text{F}$: $(5.4 \times 10^{-11}\,\text{C}) = (6.0\,\mu\text{F})V \quad V = (5.4 \times 10^{-11}\,\text{C}) \div (6.0 \times 10^{-12}\,\text{F}) = 9.0\,\text{V}$

9 μF: both have same $V = (12\,\text{V}) - (9.0\,\text{V}) = 3.0\,\text{V}$

3 $\frac{1}{2}CV^2$

a $\frac{1}{2}(2500 \times 10^{-6}\,\text{F})(1.5\,\text{V})^2 = 2.8 \times 10^{-3}\,\text{J}$

b $\frac{1}{2}(2500 \times 10^{-6}\,\text{F})(9.0\,\text{V})^2 = 0.10\,\text{J}$

c $\frac{1}{2}(2500 \times 10^{-6}\,\text{F})(12\,\text{V})^2 = 0.18\,\text{J}$

4 $V = 150\,\text{V}$ $Q = (4.5 \times 10^{-6}\,\text{C})$ $W = \frac{1}{2}QV = \frac{1}{2}(4.5 \times 10^{-6}\,\text{C})(150\,\text{V}) = 3.4 \times 10^{-3}\,\text{J}$

5 For the 6 μF, $E = \frac{1}{2}QV = 0.5 \times 5.4 \times 10^{-5} \times 9\,\text{J} = 2.43 \times 10^{-4}\,\text{J}$

For the 9 μF, $E = \frac{1}{2}QV = 0.5 \times 2.7 \times 10^{-5} \times 3\,\text{J} = 4.05 \times 10^{-5}\,\text{J}$

6 μF + 9 μF + 9 μF, $E = 3.24 \times 10^{-4}\,\text{J}$

Combined C, $E = \frac{1}{2}QV = 0.5 \times 54 \times 12 \times 10^{-6} = 3.24 \times 10^{-4}\,\text{J}$

RC circuits 1

Pages 84–85

1 $\tau = 47\text{s}$ so $2\tau = 94\text{s}$ $V = (12.0\text{V})/e^2 = 1.62\text{V} = 1.6\text{V}$ (2 s.f.)

Reading from graph: $V = 1.6\text{V}$ when $t = 94\text{s}$

2 a $\tau = RC = (2.5\,\text{M}\Omega)(0.40\,\mu\text{F}) = (2.5 \times 10^{6}\,\Omega)(0.40 \times 10^{-6}\,\text{F}) = 1.0\,\Omega\text{F}$

b $1\,\Omega\text{F} = 1\,\text{VA}^{-1}\text{CV}^{-1} = 1\,\text{CA}^{-1} = 1\,\text{AsA}^{-1} = 1\,\text{s}$

3 a $V = V_0 e^{-t/RC}$

$RC = (1.0 \times 10^3 \Omega)(3.2 \times 10^{-3}\,F) = 3.2\,s$

$V = (12.0\,V)\ e^{-(5.0\,s)/(3.2\,s)} = (12.0\,V)\ e^{-1.5625} = (12.0\ V)(0.2096) = 2.5\,V$

b $I = I_0 e^{-t/RC}$

$I_0 = V_0/R = (12.0\,V)/(1 \times 10^3\,\Omega) = 0.012\,A$

$I = (0.012\,A)\ e^{-(0.5\,s)/(3.2\,s)} = 0.010\,A$

c $Q = Q_0 e^{-t/RC}$

$Q_0 = CV_0 = (3200\,\mu F)(12.0V) = 0.0384C$

$Q = (0.0384C)\ e-(1.0s)/(3.2s) = (0.0384C)(0.732) = 0.028C$

RC circuits 2

Pages 86–87

1 $Q = Q_0 e^{-t/RC}$ $\quad Q = 75\%$ $\quad Q_0 = 0.75 Q_0$ $\quad RC = (12\,M\Omega)(500\,nF) = 6.0\,s$

$(0.75) Q_0 = Q_0 e^{-t/(6.0s)}$

$\ln(0.75) = -t/(6.0\,s)$

$t = -(6.0s)(-0.288) = 1.7\,s$

2 $V_0 = 1.0\,kV$ $\quad V = 10.0\,V$ $\quad t = 6.0\,s$ $\quad C = 2200\,\mu F$ $\quad R = ?$ $\quad V = V_0 e^{-t/RC}$

$(10.0V) = (1.0\,kV)e^{-t/RC}$

$e^{t/RC} = (1.0\,kV) \div (10.0\,V) = 100$

Taking natural logs: $t/RC = \ln(100)$

$$R = \frac{(6.0s)}{(2200\mu F)\ln(100)} = 592\,\Omega = 590\,\Omega\ (3\ s.f.)$$

3 Plot a graph of $\ln V/V$ against t. $\ln V/V$ values are: 1.40, 1.11, 0.81, 0.52, 0.22, −0.07

Graph is a straight line. Gradient $= -0.0147\ s^{-1}$ time constant $= RC = -1/\text{gradient} = 68\,s$

Radioactive decay

Pages 88–89

1 $\lambda t = (1.2 \times 10^{-4}\ y^{-1})(22\,920y) = 2.75$ $\quad N = (15\,000)e^{-2.75} = 960$

2 $\lambda t = (2.0 \times 10^{-4}\,s^{-1})(5 \times 60\,s) = 0.06$ $\quad 25\,000 = N_0 e^{-0.06}$

$N_0 = 25\,000\ e^{0.06} = 27\,000\ (2s.f.)$

3 $$T_{\frac{1}{2}} = \frac{\ln 2}{(1.55 \times 10^{-10}\ y^{-1})} = 4.47 \times 10^9\ y$$

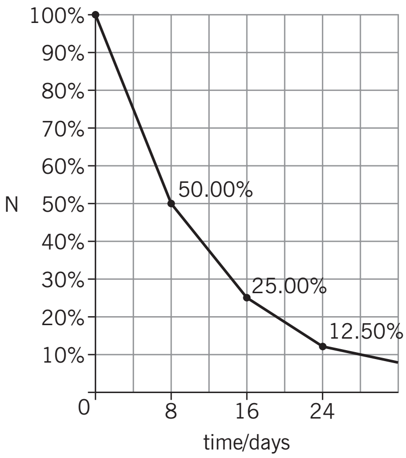
4 $$\lambda = \frac{\ln 2}{T_{½}} = \frac{\ln 2}{(6.01 \times 3600\,s)} = 3.20 \times 10^{-5}\,s^{-1}$$

5 $N_0 = 4.0g$ $\quad N = ?$ $\quad t = 10$ days $\quad T_{\frac{1}{2}} = 3.15$ days $\quad \lambda = \dfrac{\ln 2}{(3.15\ \text{days})} = 0.220\ \text{days}^{-1}$

$\lambda t = (0.220\ \text{days}^{-1})(10\ \text{days}) = 2.20$

$N = (4.0\,g)e^{-2.20} = 0.44\,g$

6



Stretch yourself

$\dfrac{\text{mass U}}{\text{mass Pb}} = \dfrac{N_u}{N_{Pb}} = 0.42$ $\quad$ for U − 238: $N = N_0 e^{-\lambda t}$ $\quad$ for Pb: $(N_0 - N)\ \dfrac{N}{(N_0 - N)} = 0.42$

$N = 0.42\,N_0 - 0.42\,N$

$1.42\,N = 0.42\,N_0$

$$N = \frac{0.42 N_0}{1.42} = 0.296\,N_0$$

$0.296 N_0 = N_0 e^{-\lambda t}$

Taking natural logs: $\ln(0.296) = -\lambda t$

$t = 1.217 \div (1.55 \times 10^{-10}\,y) = 7.9 \times 10^9\,y$

Activity

Pages 90–91

1 $\lambda = \ln(2) \div (1.6 \times 10^3\,\text{y}) = \ln(2) \div (1.6 \times 10^3 \times 365 \times 24 \times 3600\,\text{s}) = 1.37 \times 10^{-11}\,\text{s}^{-1}$
$A = -\lambda N = -(1.37 \times 10^{-11}\,\text{s}^{-1})(3.2 \times 10^{16}) = -4.4 \times 10^5\,\text{Bq}$

2 $\lambda t = \ln(2) \div (6.01\,\text{h}) \times (2\,\text{h}) = 0.231$
$A = (1.1 \times 10^4\,\text{Bq})e^{-0.231} = 8.7 \times 10^3\,\text{Bq}$

3 $\lambda t = \ln(2) \div (15\,\text{h}) \times (8\text{h}) = 0.370$
$A = (75\text{Bq})e^{-0.370} = 52\,\text{Bq}$

4 $A_0 = (9.0 \times 10^{-2}\,\text{Bq g}^{-1})(50\,\text{g}) = 4.5 \times 10^4\,\text{Bq}$ $\lambda = \ln(2) \div (5730\,\text{y}) = 1.21 \times 10^{-4}\,\text{y}^{-1}$
$A = A_0 e^{-\lambda t}$ $\ln(A/A_0) = -\lambda t$ $\ln(A_0/A) = \lambda t$
$t = \ln[(4.5 \times 10^4\,\text{Bq}) \div (1.2 \times 10^3\,\text{Bq})] \div (1.21 \times 10^{-4}\,\text{y}^{-1}) = 29\,900\,\text{y} = 3.0 \times 10^4\,\text{y}$

Stretch yourself

X: $T_{\frac{1}{2}} = 100\text{y}$ Y: $T_{\frac{1}{2}} = 10\text{y}$ $A_{X0} = A_{Y0} = A_0$ $\lambda_X = \ln(2) \div (100\,\text{y})$ $\lambda_Y = \ln(2) \div (10\,\text{y})$

a After 6 months: $\lambda_X t = [\ln(2) \div (100\,\text{y})] \times (0.5\,\text{y}) = 0.00347$
$\lambda_Y t = [\ln(2) \div (10\,\text{y})] \times (0.5\,\text{y}) = 0.0347$
$\frac{A_X}{A_Y} = \frac{e^{-0.00347}}{e^{-0.0347}} = e^{(0.0347\,-\,0.00347)} = 1.03$ so the activity of X is 1.03 times the activity of Y

b After 6 years: $\lambda_X t = [\ln(2) \div (100\,\text{y})] \times (6\,\text{y}) = 0.0416$
$\lambda_Y t = [\ln(2) \div (10\,\text{y})] \times (6\,\text{y}) = 0.416$
$A_X/A_Y = e^{(0.416\,-\,0.0416)} = 1.5$ so the activity of X is 1.5 times the activity of Y

5 Values of ln A: 18.22, 18.10, 17.99, 17.86, 17.74
$-\lambda = (17.5 - 18.22) \div (3.0 - 0)\,\text{h} = 0.24\,\text{h}^{-1}$
$T_{\frac{1}{2}} = \ln\frac{2}{0.24}\text{h}^{-1} = 2\,\text{h}\ 53\ \text{minutes}$

Nuclear reactions

Pages 92–93

1 a ${}^{240}_{94}\text{Pu} \rightarrow {}^{236}_{92}\text{U} + {}^{4}_{2}\text{He}$

b ${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + {}^{0}_{0}$

2 ${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{p}$ ($A = 1$ $Z = 1$ is a proton)

3 a p: $\frac{1.673 \times 10^{-27}\,\text{kg}}{1.661 \times 10^{-27}\,\text{kg u}^{-1}} = 1.007\,\text{u}$

b n: $\frac{1.675 \times 10^{-27}\,\text{kg}}{1.661 \times 10^{-27}\,\text{kg u}^{-1}} = 1.008\,\text{u}$

c e: $\frac{9.11 \times 10^{-31}\,\text{kg}}{1.661 \times 10^{-27}\,\text{kg u}^{-1}} = 5.485 \times 10^{-4}\,\text{u}$

4 a ${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$
Po nucleus 209.937 u
Pb nucleus 205.929 u
Alpha = 4.002 u

b $\Delta m = 209.937\,\text{u} - (205.929\,\text{u} + 4.002\,\text{u}) = 0.006\,\text{u}$

c $\Delta E = 5.6\,\text{MeV}$

Fission and fusion

Pages 94–95

1 a $2 \times 1.007 + 2 \times 1.008 = 4.03\,\text{u}$

b $\Delta m = 4.03\,\text{u} - 4.00\,\text{u} = 0.030\,\text{u} = 4.983 \times 10^{-29}\,\text{kg}$

c In MeV $= \frac{0.030\,\text{u} \times (1.661 \times 10^{-27}\,\text{kg u}^{-1}) \times (3.0 \times 10^8\,\text{m s}^{-1})^2}{1 \times 10^6 \times (1.60 \times 10^{-19}\text{J eV}^{-1})} = 28\,\text{MeV}$

2 a 26p + 30n $m = 26(1.007\,\text{u}) + 30(1.008\,\text{u}) = 56.422$ $\Delta m = 56.422 - 55.935 = 0.487\,\text{u}$

b $\Delta E = 0.487\,\text{u} \times 931.5\,\text{MeV u}^{-1} = 453.6\,\text{MeV} = 454\,\text{MeV}$
or $\Delta E = \frac{(0.487\,\text{u}) \times (1.661 \times 10^{-27}) \times (3.0 \times 10^8\,\text{m s}^{-1})^2}{1 \times 10^6 \times (1.60 \times 10^{-19}\,\text{J eV}^{-1})} = 455\,\text{MeV}$
(The difference in value is due to the number of significant figures for the constants c and e.)

c Per nucleon $= \Delta E/A = (453.6\,\text{MeV}) \div (56) = 8.1\,\text{MeV}$ (to get the accepted value of 8.79 MeV per nucleon, the mass of the neutron and proton should be to more s.f., also the mass of the Fe nucleus)

3 **a** Initial mass = (235.0439 u) + (1.0087 u)
Final mass = (147.9322 u) + 3(1.0087 u)
Δm = (236.0526 u) − (235.8739 u) = 0.1787 u
ΔE = (0.1787 u) × (931.5 MeV u^{-1}) = 166 MeV
b Initial mass = (14.0031 u) + (1.0078 u)
Final mass = (15.0031 u) + 0
Δm = (15.0031 u) − (15.0031 u) = 7.8 × 10^{-3} u
ΔE = (7.8 × 10^{-3} u) × (931.5MeV u^{-1}) = 7.27 MeV

Fundamental particles 1

Pages 96–97

1 $p \rightarrow n + e^{+} + \nu_e$
charge: (+1) → (0) + (+1) + (0) conserved
lepton no: (0) → (0) + (−1) + (+1) conserved
2 $\mu^{-} \rightarrow e^{-} + \bar{\nu}_e + \nu_{\mu}$
charge: (−1) → (−1) + (0) + (0) conserved
lepton no: (+1) → (+1) + (−1) + (+1) conserved
3 uud is a proton. proton has charge +1 lepton no. = 0 baryon no. = +1
uud has charge (+2/3) + (+2/3) + (−1/3) = (+1) conserved
lepton no. 0 + 0 + 0 = 0 conserved
baryon no. (+1/3) + (+1/3) + (+1/3) = (+1) conserved
$\bar{u}\bar{u}\bar{d}$ is an anti−proton. Anti−proton has charge −1 lepton no. = 0 baryon no. = −1
$\bar{u}\bar{u}\bar{d}$ has charge (−2/3) + (−2/3) + (+1/3) = (−1) conserved
lepton no. 0 + 0 + 0 = 0 conserved
baryon no. (−1/3) + (−1/3) + (−1/3) = (−1) conserved
$\bar{u}\bar{d}\bar{d}$ is an anti–neutron. Anti–neutron has charge 0 lepton no. = 0 baryon no. = −1
$\bar{u}\bar{d}\bar{d}$ has charge (−2/3) + (+1/3) + (+1/3) = (0) conserved
lepton no. 0 + 0 + 0 = 0 conserved
baryon no. (−1/3) + (−1/3) + (−1/3) = (−1) conserved
4 **a, b, c**

Particle	Quarks	Charge/e	Strangeness	Lepton number	Baryon number
Ω^{-}	sss	−1	−3	0	1
Λ^{0}	uds	0	−1	0	1
Σ^{+}	uus	+1	−1	0	1
Σ^{0}	uds	0	−1	0	1
Σ^{-}	dds	−1	−1	0	1

Fundamental particles 2

Pages 98–99

1 **a** $d\bar{s}$ $\bar{d}s$ $u\bar{s}$ $\bar{u}s$
b charge: $d\bar{s}$ (−1/3) + (+1/3) = 0 which is K^{0}
$\bar{d}s$ (+1/3) + (−1/3) = 0 which is K^{0}
$u\bar{s}$ (+2/3) + (+1/3) = +1 which is K^{+}
$\bar{u}s$ (−2/3) + (−1/3) = −1 which is K^{-}
c strangeness: $d\bar{s}$ = +1 $\bar{d}s$ = −1 $u\bar{s}$ +1 $\bar{u}s$ −1
2 $\pi^{0} \rightarrow e^{+} + e^{-} + \gamma + \gamma$
Charge: 0 → (+1) + (−1) + (0) + (0) = 0 conserved
Lepton no. 0 → (−1) + (+1) + (0) + (0) = 0 conserved
Baryon no. 0 → (0) + (0) + (0) + (0) = 0 conserved
3 **a** $K^{-} \rightarrow \pi^{?} + e^{-} + \bar{\nu}_e$
Lepton no. (0) → (0) + (+1) + (−1) = 0 conserved
Charge: (−1) → (0) + (−1) + (0) charge must be 0 in order to be conserved
So $K^{-} \rightarrow \pi^{0} + e^{-} + \bar{\nu}_e$
b charge on pion = 0
c Strangeness: (−1) → (0) + (0) + (0) so strangeness is not conserved.
4 Energy of e^{+} and e^{-} = 0.51 MeV + 0.51 MeV + 0.25MeV + 0.25 MeV = 1.52 MeV
Energy of photon = 1.52 MeV

5 a $\dfrac{(1.673 \times 10^{-27}\,\text{kg}) \times (3.0 \times 10^{8}\,\text{m s}^{-1})^2}{1 \times 10^{6} \times (1.60 \times 10^{-19}\text{J eV}^{-1})} = 941\ \text{MeV/c}^2$

b $\dfrac{(1.675 \times 10^{-27}\,\text{kg}) \times (3.0 \times 10^{8}\,\text{m s}^{-1})^2}{1 \times 10^{6} \times (1.60 \times 10^{-19}\text{J eV}^{-1})} = 942\ \text{MeV/c}^2$

6 $\dfrac{(1.661 \times 10^{-27}\,\text{kg}) \times (3.0 \times 10^{8}\,\text{m s}^{-1})^2}{1 \times 10^{6} \times (1.60 \times 10^{-19}\text{J eV}^{-1})} = 934\ \text{MeV/c}^2$

Note that the standard value of 931.5 MeV/c^2 requires a larger number of significant figures: u = 1.6605×10^{-27} kg and $c = 2.9979 \times 10^{8}\,\text{m s}^{-1}$ and $e = 1.6021 \times 10^{-19}$ C.

7 Total energy $E = 2.5\,\text{MeV} = 2m_0c^2 + E_K$ (E_K = total kinetic energy of both particles) = 1.02MeV + E_K
E_K = 2.5MeV − 1.02 MeV = 1.48 MeV = 1.5 MeV (2 s.f.)

8 Total energy E = 2(0.51 +5.0) MeV = 2(5.51 MeV)
2 photons each of energy 0.5 × 2(5.51 MeV) = 5.51 MeV (note that it is important to show that you know there are two particles producing 2 photons – using energy of one particle to get the energy of one photon may lose you marks)
$\lambda = (6.63 \times 10^{-34}\,\text{J s})(3.0 \times 10^{8}\,\text{m s}^{-1})(5.51 \times 10^{6}\,\text{eV})(1.6 \times 10^{-19}\,\text{J eV}^{-1})$
$\lambda = 2.3 \times 10^{-13}\,\text{m}$

Stretch yourself

Minimum energy $= \dfrac{2(1.9 \times 10^{-28}\,\text{kg}) \times (3.0 \times 10^{8}\,\text{m s}^{-1})^2}{1 \times 10^{6} \times (1.60 \times 10^{-19}\text{J eV}^{-1})} = 214\,\text{MeV} = 210\,\text{MeV (2 s.f.)}$

X-rays

Pages 100–101

1 $A = \pi r^2 \quad P = IA = (4.5 \times 10^{8}\,\text{W m}^{-2})\pi\,(2.5 \times 10^{-2}\,\text{m})^2 = 8.8 \times 10^{5}\,\text{W}$

2 $E = Pt = IAt \quad A = \dfrac{E}{It} = \dfrac{(4.6 \times 10^{6}\,\text{J})}{(6.0 \times 10^{8}\,\text{W m}^{-2})(2.0\,\text{s})} = 3.83 \times 10^{-3}\,\text{m}^2$

$A = \pi r^2\ r = \sqrt{[(3.83 \times 10^{-3}\,\text{m}^2) \div \pi]} = 0.035\,\text{m}\ (= 3.5\text{cm})$

3 $A = 4\pi r^2\ P = IA = (6.0 \times 10^{-7}\,\text{W m}^{-2})4\pi\,(1.5 \times 10^{11}\,\text{m})^2 = 1.7 \times 10^{17}\,\text{W}$

4 $I = I_0 e^{-\mu x}\ \mu x = (1.5\,\text{mm}^{-1})(3.0\,\text{mm}) = 4.5$
$I = (4.8 \times 10^{8}\,\text{W m}^{-2})e^{4.5} = 5.3 \times 10^{6}\,\text{W m}^{-2}$

5 $I = 12\%\ I_0 = 0.12\ I_0\ 0.12\ I_0 = I_0 e^{-\mu x}\ 0.12 = e^{-\mu x}$
Taking natural logs. ln (0.12) = $-\mu x = -\mu(4.0\,\text{mm})$
$\mu = 0.530\,\text{mm}^{-1}$
when $I = 20\%\ I_0$ ln (0.20) = $-\mu x = -(0.530\,\text{mm}^{-1})x$
$x = 3.0\,\text{mm}$

Red shift

Pages 102–103

1 a $z = 0.001\ v = zc\ v = (-0.001)(3.0 \times 10^{8}\,\text{m s}^{-1}) = 3.0 \times 10^{5}\,\text{m s}^{-1}$ towards Earth.

b $\lambda = 589.59\,\text{nm} \quad \Delta\lambda/\lambda = -0.001 \quad \Delta\lambda = (-0.001)(589.59\,\text{nm}) = 589.00\,\text{nm}$
shifted $\lambda = \lambda + (-0.58959\,\text{nm}) = (589.59-0.58959)\text{nm} = 598.00\,\text{nm}$

2 It is moving away (because red shift means to longer wavelength).
$\Delta\lambda/\lambda = 12/100 = v/c$
$v = 0.12 \times (3.0 \times 10^{8}\,\text{m s}^{-1}) = 3.6 \times 10^{7}\,\text{m s}^{-1}$

3 $z = 0.064$
$v = zc = (0.064)(3.0 \times 10^{8}\,\text{m s}^{-1}) = 1.92 \times 10^{7}\,\text{m s}^{-1}$

The expanding Universe

Pages 104–105

1 $1\text{ly} = (3.0 \times 10^{8}\,\text{m s}^{-1})(365.25 \times 24 \times 60 \times 60\,\text{s}) = 9.47 \times 10^{15}\,\text{m}$

2 722.9 ly = (722.9 ly)÷(3.26 ly pc^{-1}) = 222 pc

3 $(1.32 \times 10^{10}\,\text{y})\dfrac{(74.3)}{(74.3 + 2.1)} = 1.28 \times 10^{10}\,\text{y}$

$(1.32 \times 10^{10}\,\text{y})\dfrac{(74.3)}{(74.3-2.1)} = 1.36 \times 10^{10}\,\text{y}$

4 $v = zc \quad v = H_0 d \quad zc = H_0 d \quad d = zc/H_0 = \dfrac{(8.6)(3.0 \times 10^{5}\,\text{km s}^{-1})}{(74.3\,\text{km s}^{-1}\text{Mpc}^{-1})} = 3.5 \times 10^{4}\,\text{Mpc}$

Stars

Pages 106–107

1 $\lambda_{max} = \dfrac{(2.89 \times 10^{-3}\,\text{m K})}{(5800\,\text{K})} = 5.0 \times 10^{-7}\,\text{m}$

2 **a** $\lambda_{max} = \dfrac{(2.89 \times 10^{-3}\,\text{m K})}{(3500\,\text{K})} = 8.3 \times 10^{-7}\,\text{m}$ is in the IR range so it would appear red.

b $r = 8.2 \times 10^{8}\,\text{m} \quad L = 4\pi r^2\,\sigma T^4$

$L = 4\pi(8.2 \times 10^{8}\,\text{m})^2(5.67 \times 10^{-8}\,\text{W m}^{-2}\,\text{K}^{-4})(3500\,\text{K})^4 = 7.2 \times 10^{25}\,\text{W}$

Stretch yourself

$\lambda_{max} = 670\,\text{nm} = \dfrac{(2.89 \times 10^{-3}\,\text{m K})}{T}$

a $T = \dfrac{(2.89 \times 10^{-3}\,\text{m K})}{(670 \times 10^{-9}\,\text{m})} = 4.31 \times 10^{3}\,\text{K}$

b $L_A = 170 L_{Sun} \quad L = 4\pi r^2\,\sigma T^4 \quad \dfrac{L_A}{L_{Sun}} = 170 = \dfrac{4\pi {r_A}^2\,\sigma {T_A}^4}{4\pi {r_{Sun}}^2\,\sigma {T_{Sun}}^4} = \dfrac{{r_A}^2 {T_A}^4}{{r_{Sun}}^2 {T_{Sun}}^4}$

${r_A}^2 = \dfrac{170\,{r_{Sun}}^2 {T_{Sun}}^4}{{T_A}^4} = \dfrac{170(6.96 \times 10^{8}\,\text{m})^2(5800\,\text{K})^4}{(4.31 \times 10^{4}\,\text{K})^4} = 2.700 \times 10^{20}\,\text{m}^2$

$r_A = 1.6 \times 10^{10}\,\text{m}$

3

	Hot and bright X Rigel	Cool and bright X Arcturus
$\dfrac{L}{L_{Sun}}$	Hot and dim	X Sun Cool and dim

T/K