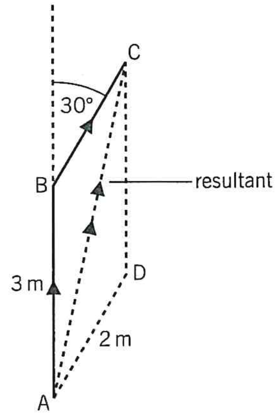
Forces

Resultant forces

Forces are vectors. When vectors are combined their direction must be taken into account. This diagram shows that walking 3 m from A to B and then turning through 30° and walking 2 m to C has the same effect as walking directly from A to C. $\overrightarrow{AC}$ is the **resultant** vector, denoted by the double arrowhead.

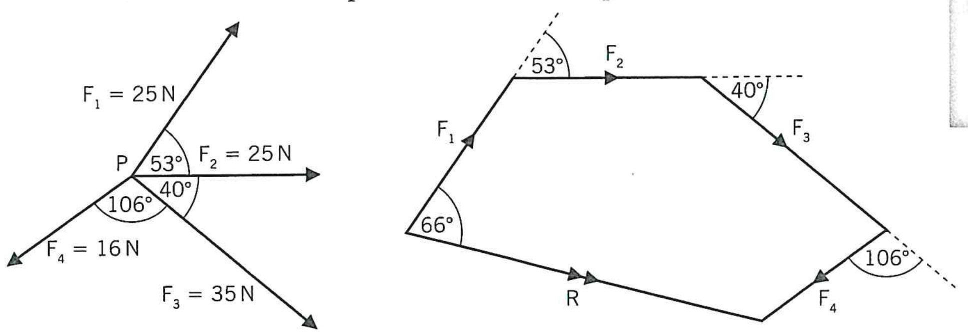


To combine forces, we can draw a similar diagram where the lengths of the sides represent the magnitude of the force (for example 30 N and 20 N). The third side of the triangle shows us the magnitude and direction of the resultant force. A careful drawing of a scale diagram allows us to measure these. Notice that if the vectors are combined by drawing them in the opposite order, $\overrightarrow{AD}$ and $\overrightarrow{DC}$, these are the other two sides of parallelogram and give the same resultant.

WORKED EXAMPLE: POLYGON OF FORCES

This result can be extended for many forces. Instead of a triangle we can draw a **polygon of forces**. This polygon is used to find the resultant of the four forces at P. Notice that each force starts from where the previous one ended, and the resultant is the direct route from the initial start point to the final end point.

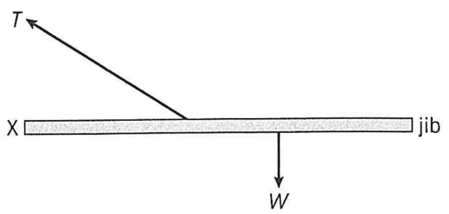
> **REMEMBER:** If a set of forces is in equilibrium, then the last force in the polygon will join to the first force in the polygon and the resultant is zero.



R = 53 N 66° clockwise from F_1

PRACTICE QUESTIONS

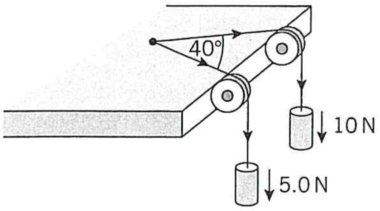
1 There are three forces on the jib of a tower crane. The tension in the cable T, the weight W, and a third force P acting at X.



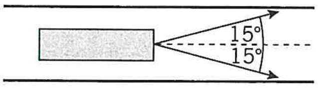
The crane is in equilibrium. Sketch the triangle of forces.

2 For each of these situations draw a triangle or polygon of forces to determine the resultant force:

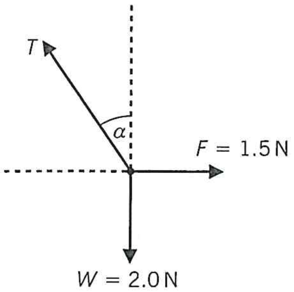
a



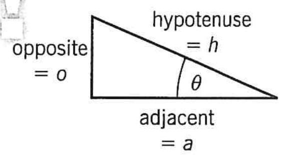
b Two forces of 5 kN towing a boat



3 These three forces are in equilibrium. Draw a triangle of forces to find T and α.



REMEMBER:



Pythagoras's theorem: For a right-angled triangle as shown: $h^2 = o^2 + a^2$

$\sin\theta = \frac{o}{h}$, $\cos\theta = \frac{a}{h}$

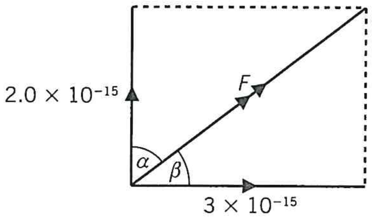
$\tan\theta = \frac{o}{a}$ (sohcahtoa)

Calculating resultants

When two forces are acting at right angles the resultant can be calculated using Pythagoras's theorem and trig functions, sine, cosine and tangent.

WORKED EXAMPLE

A sub-atomic particle experiences two forces at right angles, one of 2.0×10^{-15} N and the other 3.0×10^{-15} N.



$\alpha = \tan^{-1}\frac{3}{2} = 56°$

$\beta = \tan^{-1}\frac{2}{3} = 34°$

The resultant is represented by F.

$F^2 = (2.0 \times 10^{-15}\,\text{N})^2 + (3.0 \times 10^{-15}\,\text{N})^2$

$F = \sqrt{(4.0 + 9.0)} \times 10^{-15}\,\text{N}$

$F = 3.6 \times 10^{-15}\,\text{N}$

The angle is calculated using either $\tan\alpha$ or $\tan\beta$, remember to state, or show on the diagram, which angle you use. (This diagram shows both, but you only need to calculate one.)

PRACTICE QUESTION

4 Find the resultant force for these pairs of forces at right angles:

a 3.0 N and 4.0 N   b 5.0 N and 12.0 N