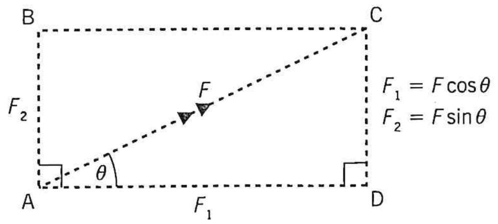
Resolving forces

Perpendicular forces

Just as we can find the **resultant** of several forces, we can replace a force with components that have the same effect, when combined, as the original force. This called **resolving** a force. Resolving a force into two components at right angles is the key to solving many problems.

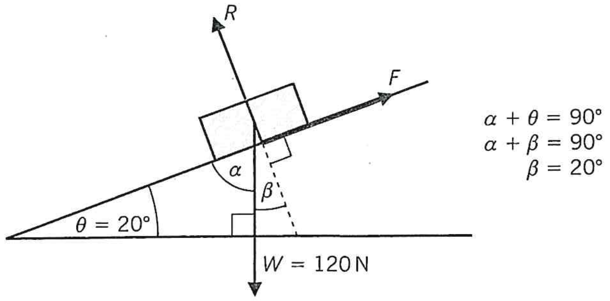
This diagram shows that when a force F is resolved into two components at right angles the components are:

$F_1 = F\cos\theta$ and $F_2 = F\sin\theta$ where θ is the angle between F and F_1.



WORKED EXAMPLE

A block of wood of weight, W, is placed on a slope. The friction, F, is large enough to prevent it slipping down.



REMEMBER: The angles of a triangle add up to 180°.
When two straight lines intersect, the opposite angles are always equal (because a straight line is 180°).
$\cos(90° - \theta) = \sin\theta$

The forces and motion parallel to the slope are independent from the forces and motion perpendicular to the slope. Resolving in these two directions simplifies the equations, as the friction has no effect perpendicular to the slope and the normal reaction R has no effect parallel to the slope.

The three forces are in equilibrium so:

Resolving perpendicular to the slope: $R = W\cos\beta = W\cos\theta$

$R = (120\ \text{N})\cos 20° = 110\ \text{N}$

Resolving parallel to the slope: $F = W\cos\alpha = W\sin\theta$

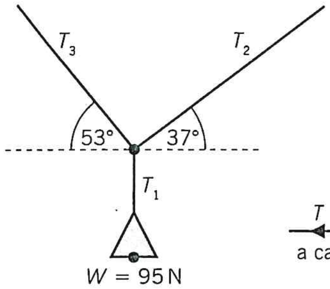
$F = (120\ \text{N})\sin 20° = 41\ \text{N}$

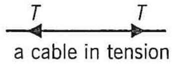
PRACTICE QUESTIONS

1. A force of 550 N is applied to a box at an angle of 30° to the horizontal. Calculate the horizontal and vertical components of the force.
2. Calculate the normal reaction and the friction for a box of weight 85 N in equilibrium on a slope of angle 15°.

WORKED EXAMPLE

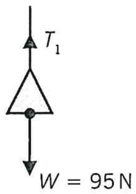
A lamp hangs from three cables tied as shown.

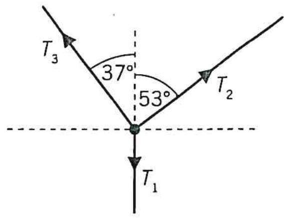




A string or cable can be in tension, but not in compression. If it is in tension, the force pulls on the objects it is attached to. The tension is the same at every point in the cable, or the cable would break, so it is always the same at both ends.

The lamp and the knot in the cable are both in equilibrium. Draw free body force diagrams for the lamp and the knot in the cable:





For the lamp: $T_1 = 95\text{ N}$ (1)
For the join in the cables: $\rightarrow$ (resolving horizontally)
$T_3 \cos 53° = T_2 \cos 37°$ (2)

$\uparrow$ (resolving vertically) $T_1 = T_3 \sin 53° + T_2 \sin 37°$ (3)

From (2): $T_3 = \dfrac{T_2 \cos 37°}{\cos 53°} = 1.33\ T_2$

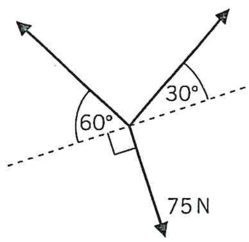
From (1) and (3) substitute for T_1 and T_3:
$95\text{ N} = 1.33\ T_2 \sin 53° + T_2 \sin 37°$
$95\text{N} = T_2\ (1.33 \sin 53° + \sin 37°) = T_2\ 1.66$

$T_2 = \dfrac{95\text{ N}}{1.66} = 57\text{ N}$

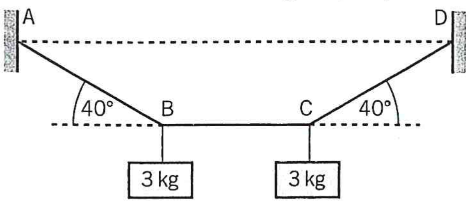
From (2) $T_3 = 1.33 \times 57\text{ N} = 76\text{ N}$

PRACTICE QUESTIONS

3 The three strings in the diagram are in tension and in equilibrium. Calculate the tension in each string.



4 Two masses are supported by three strings. BC is horizontal. What is the tension in string AB, BC, and CD?



5 A cable, parallel to a slope 30° to the horizontal, pulls a block up the slope at a steady speed. The block weighs 65 N and the friction with the slope is 12 N. What is the tension in the cable, and the normal reaction force?