Units and dimensions

Base and derived SI units

Units are defined so that, for example, every scientist who measures a mass in kilograms uses the same size for the kilogram and gets the same value for the mass. Scientific measurement depends on standard units. Every measurement must give the unit to have any meaning. You should know the correct unit for physical quantities.

EXAMPLES OF QUANTITIES AND UNITS

Base units

Physical quantity	Unit	Symbol	Physical quantity	Unit	Symbol
length	metre	m	electric current	ampere	A
mass	kilogram	kg	temperature difference	kelvin	K
time	second	s	the amount of substance	mole	mol

Derived units

Example: Speed

Speed is defined as $\frac{\text{distance travelled}}{\text{time taken}}$.

If a car travels 2 metres in 2 seconds its speed is $\frac{2\text{ metres}}{2\text{ seconds}} = 1\frac{\text{m}}{\text{s}}$.

REMEMBER: Avoid unit errors: a numerical answer is incorrect if the unit is missing.

This defines the SI unit of speed to be 1 metre per second, which is written $1\,\text{m}\,\text{s}^{-1}$ $\left(\text{where } \text{s}^{-1} = \frac{1}{\text{s}}\right)$.

PRACTICE QUESTION

1 Complete this table with the missing units and symbols.

Physical quantity	Equation used to derive unit	Unit	Symbol and name (if there is one)
frequency	period^{-1}	s^{-1}	Hz hertz
volume	length^3		–
acceleration	velocity ÷ time		–
force	mass × acceleration	$\text{kg}\,\text{m}\,\text{s}^{-2}$	
work and energy	force × distance		J joule
voltage	energy ÷ electric charge	$\text{J}\,\text{C}^{-1}$	
electrical resistance		$\text{V}\,\text{A}^{-1}$	

Using base units to check equations and constants

The physical quantities that are measured in base units are called dimensions. Base units can be used to check if an equation is dimensionally correct. They can also be used to check the units of a constant.

CHECKING EQUATIONS

To check whether the equation Kinetic energy $= \frac{1}{2} m v^2$ is dimensionally correct.

Left-hand side of equation (LHS): J = N m = kg m s^{-2} × m = kg $m^2 s^{-2}$.

Right-hand side (RHS): kg × $(m\,s^{-1})^2$ = kg $m^2 s^{-2}$. (Notice that the constant $\frac{1}{2}$ has no units.)

The two sides are the same so the equation is dimensionally correct. We can't tell that there is $\frac{1}{2}$ in the equation so we can't tell that the equation is correct, only that it is dimensionally correct.

PRACTICE QUESTIONS

2 Use base units to show the equation $Q = It$ for electric charge passing a point in time t when the electric current is I, is dimensionally correct.

3 Use base units to show that the equation $P = IV$ is dimensionally correct, where I is electric current, V is voltage and P is power measured in watts (W) 1 W = 1 J s^{-1}.

UNITS FOR CONSTANTS

The Planck constant h is used in the equation that tells us the energy of a photon: energy $E = hf$ where f is the frequency of the radiation. We can use this equation to find the units for h.

LHS: J

RHS: (units of h) s^{-1}

Units of h = J s

PRACTICE QUESTIONS

4 The Earth's gravitational field strength $g = 9.81$ N kg^{-1} is also sometimes given as the acceleration due to gravity $g = 9.81$ m s^{-2}. Show that these units are equivalent.

5 Newton's law of gravitation says that the force F between two masses is $F = G\frac{m\,M}{r^2}$ where m and M are two masses and r is the distance between them. Find the units of the gravitational constant G.

REMEMBER: It's very important to use upper case letters and lower case letters correctly.
For example:
N for newton and m for metre.

STRETCH YOURSELF

Show that the formula for the period of a pendulum

$T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$ is dimensionally correct.