Motion 1

Vectors and scalars

Scalar quantities have magnitude but no direction. Vector quantities have magnitude and direction.

WORKED EXAMPLE: SPEED, VELOCITY AND ACCELERATION

Speed is measured in metres per second, per means 'in each.'
We can work out the average speed by measuring the distance travelled and dividing by the time taken. Speed is a scalar quantity this means it will always be positive. Distance and time are also scalars. They are always positive.

Velocity is a vector quantity. It is speed in a certain direction. When calculating velocity, v, we need to know the **displacement**, s. Displacement is the distance travelled in a certain direction. Displacement is a vector. For vectors, the opposite direction is the negative direction. Other directions are sometimes indicated by giving angles.

Acceleration is calculated from the change in velocity, Δv, divided by the time taken, Δt. (Δ is the symbol 'delta' and is used to mean 'a change in'.) Acceleration is the rate of change of velocity, not speed. This means that it is a vector. If an object is moving in a circle at constant speed, its direction is changing, so its velocity is changing, so it is accelerating.

PRACTICE QUESTIONS

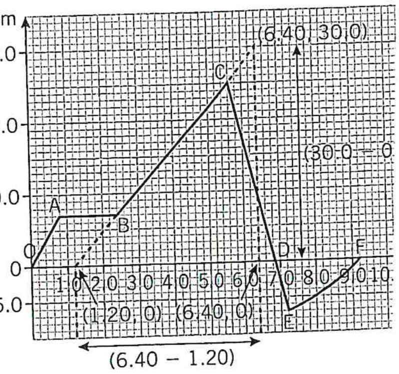
1. Divide these quantities into vectors and scalars: density, electric charge, electrical resistance, energy, field strength, force, friction, frequency, mass, momentum, power, voltage, volume, weight, work done.
2. Divide this data into vectors and scalars: $3\,\text{m}\,\text{s}^{-1}$, $+20\,\text{m}\,\text{s}^{-1}$, 100 m NE, 50 km, −5 cm, 10 km S 30° W.

STRETCH YOURSELF

Work done = force (N) × distance moved in direction of the force (m).
It is measured in joules, 1 J = 1 N m
A moment = force (N) × perpendicular distance from the pivot (m). It is measured in N m.
Explain whether these quantities and units are the same.

WORKED EXAMPLE: DISPLACEMENT-TIME GRAPHS

This graph shows the horizontal displacement of the cursor from a point on a monitor screen.



REMEMBER: Always read values to the nearest half, or quarter, of a small square – not to the nearest square.

Points to note are: OA constant velocity, AB stationary, BC constant but slower velocity, CE constant, faster, negative velocity. At D the cursor passes through the starting point and continues moving away from it, between E and F the velocity is positive and the cursor is accelerating towards the starting point – the displacement is negative until F when it is zero again.

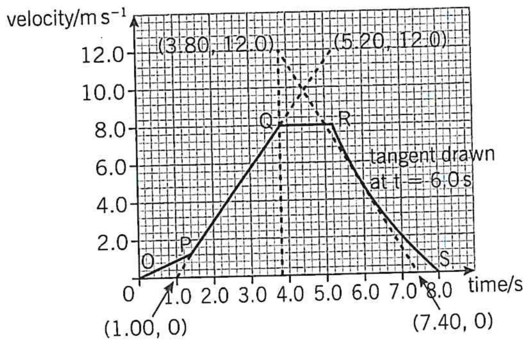
The velocity between B and C can be found from the gradient of the graph. To calculate the gradient, always draw the largest triangle possible to reduce uncertainties:

Gradient $= \dfrac{(30.0 - 0.0)\ \text{cm}}{(6.40 - 1.20)\ \text{s}} = 5.77\ \text{cm s}^{-1}$. Don't forget the units, or to work out a reasonable number of significant figures based on how many you can read from the graph.

Velocity between BC $= 5.77\ \text{cm s}^{-1}$ away from the point in the positive direction.

WORKED EXAMPLE: VELOCITY–TIME GRAPHS

This graph shows how the velocity of an object changes with time.



Points to note from this graph are: the object was stationary at O, accelerated with constant acceleration to P, PQ faster constant acceleration, QR constant velocity, RS slowing down, but not with uniform deceleration, stopping at S. The velocity was always positive so the object was moving in the same direction throughout.

The acceleration between P and Q can be found from the gradient of the graph.

Gradient $= \dfrac{(12.0 - 0.0)\ \text{m s}^{-1}}{(5.20 - 1.00)\ \text{s}} = 2.86\ \text{m s}^{-2}$ acceleration $= 2.86\ \text{m s}^{-2}$

The acceleration between R and S is negative. At any time it is equal to the gradient of the curve at that time. To find the acceleration at $t = 6.0$ s, we need to draw a tangent to the curve at the point where $t = 6.0$ s and find the gradient of the tangent. The tangent is the line that 'just touches' the curve at that point.

Gradient $= \dfrac{(0.0 - 12.0)\ \text{m s}^{-1}}{(7.45 - 3.8)\ \text{s}} = -3.29\ \text{m s}^{-2}$ acceleration $= -3.29\ \text{m s}^{-2}$

PRACTICE QUESTIONS

3 For the displacement–time graph on page 22, calculate the velocity between:

a O and A

b C and E.

4 For the velocity–time graph above, calculate the acceleration:

a between O and P

b at $t = 7.0$ s

Motion 2

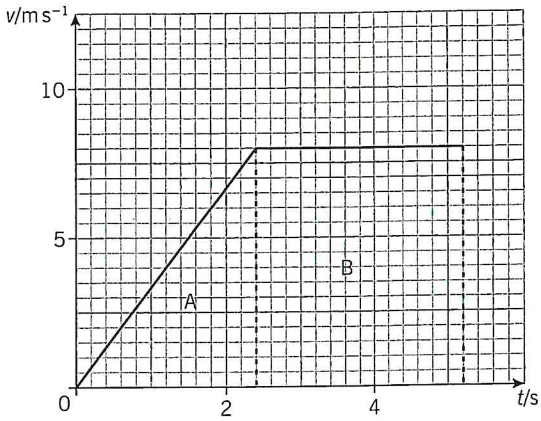
The distance travelled

The distance travelled by an object at constant velocity is equal to the velocity × time of travel. On a velocity–time graph, this is the area of the rectangle between the line and the time axis. For objects travelling with changing velocity, you can imagine that the area can be calculated from lots of very thin rectangles, with widths equal to very small time intervals, and height equal to the average velocity during that time interval.
This leads to the general conclusion that:

> the area under the curve of a velocity–time graph is equal to the distance travelled.

Areas can be calculated using the formulae for the area of a rectangle, a triangle or trapezium, or by 'counting the squares' under a curve.

WORKED EXAMPLE: TRIANGLES AND RECTANGLES



The area of the triangle marked A

$= \frac{1}{2}$ base × height

$= 0.5 \times 2.40\,\text{s} \times 8.0\,\text{m s}^{-1} = 9.6\,\text{m}^2$

The area of the rectangle marked B

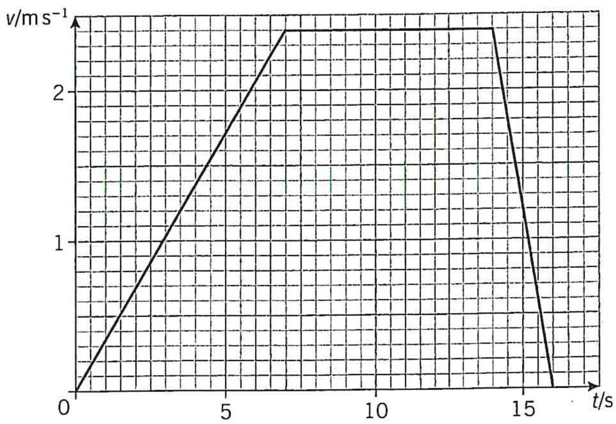
$=$ base × height

$= (5.20 - 2.40)\,\text{s} \times 8.0\,\text{m s}^{-1} = 22.4\,\text{m}^2$

The total distance travelled $= 9.6\,\text{m} + 22.4 = 32.0\,\text{m}$

$= 32\,\text{m}$ (2 s.f.)

WORKED EXAMPLE: TRAPEZIUM



The area of the trapezium

$= \frac{1}{2}$ (sum of parallel sides) × height

$= 0.5 \times (7.0 + 16.0)\,\text{s} \times 2.4\,\text{m s}^{-1}$

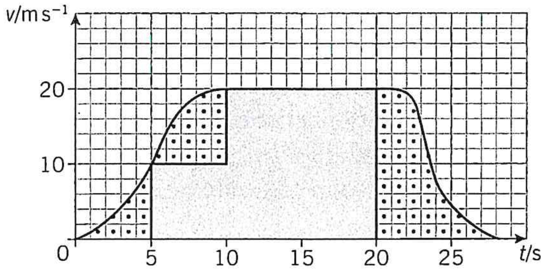
$= 27.6\,\text{m}^2$

The total distance travelled $= 28\,\text{m}$ (2 s.f.)

PRACTICE QUESTIONS

1. Use the formula for the area of a trapezium to check the area of the example above calculated by triangle and rectangle.
2. Use the triangle and rectangle formulae to check the area of the example above calculated by trapezium.

WORKED EXAMPLE: COUNTING SQUARES



Each small square has an area of $1.0\,\mathrm{s} \times 2.0\,\mathrm{m\,s^{-1}} = 2.0\,\mathrm{m^2}$. A large square ($5 \times 5 = 25$) has an area of $25 \times 2.0\,\mathrm{m} = 50\,\mathrm{m^2}$. It is important to mark off the squares as you count them to avoid missing squares or double counting.

Count all the squares that are half or more under the curve, leave out those where less than half is under the curve.

Total squares = 5 large + 69 small

Distance travelled $\approx 5 \times 50\,\mathrm{m} + 69 \times 2.0\,\mathrm{m} = 250\,\mathrm{m} + 138\,\mathrm{m}$

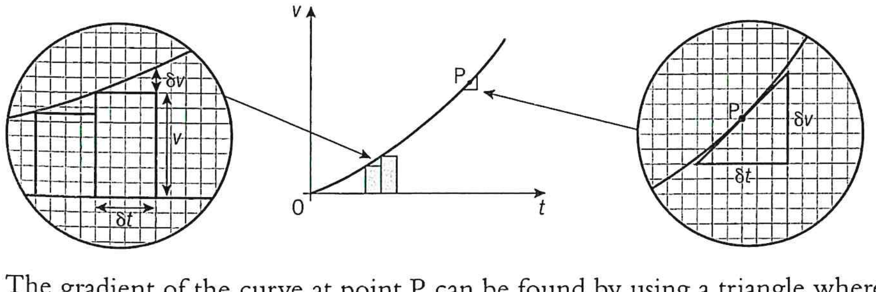
$= 388\,\mathrm{m} = 390\,\mathrm{m}$ (3 s.f.)

PRACTICE QUESTION

3 Find the distance travelled for the velocity–time graph on page 23.
(Hint: Use all three methods.)

More about curves

The area under a curve can be found by dividing it into very thin trapeziums of width δt, or sometimes Δt. (The symbol δ is also called 'delta', it means a very small change.) The total area is the sum of the areas of all the trapeziums. $\Sigma\,(v + \frac{1}{2}\,\delta v)\,\delta t$ where the symbol Σ means 'the sum of all'. As δt gets smaller and closer to zero, we write this $\delta t \rightarrow 0$, the value for the area gets closer to the true value.



The gradient of the curve at point P can be found by using a triangle where the base δt is very small. The gradient of the straight line at P is $\dfrac{\delta v}{\delta t}$.
As $\delta t \rightarrow 0$ the value for the gradient gets close to the true value.

It is then written $\dfrac{dv}{dt}$ and is called 'd v by d t' and is the gradient of a velocity–time (v–t) graph. It is the rate of change of v with t and is the acceleration at that point.

PRACTICE QUESTION

4 What is the value of $a = \dfrac{dv}{dt}$ at $t = 5.0\,\mathrm{s}$ for the three graphs in the worked examples?