Uncertainties and significant figures

Uncertainties

When a physical quantity is measured there will always be a small difference between the measured value and the true value. How important the difference is depends on the size of the measurement and the size of the uncertainty, so it is important to know this information when using data. There are several possible reasons for uncertainty in measurements, including the difficulty of taking the measurement, the precision of the measuring instrument (for example due to the size of the scale divisions), and the natural variation of the quantity being measured. The word uncertainty is generally used in preference to error, because the word error is associated with something that is wrong. Mistakes in making measurements should be avoided, not included in the uncertainty.

EXAMPLES OF UNCERTAINTIES IN MEASUREMENTS

A measurement of 4.7 g on a scale with divisions of 0.1 g means the value is closer to 4.7 g than 4.6 g or 4.8 g. If the measurement was exactly half-way between 4.7 g and 4.8 g you would round up and record 4.8 g, so 4.7 g is anything from 4.65 g up to, but not including, 4.75 g and the measurement is written 4.7 ± 0.05 g.

A length of 6.5 m measured with great care and a 10 m tape measure marked in mm could have an uncertainty of 2 mm and would be recorded as 6.500 ± 0.002 m.

The same length measured with a stick 1 m in length and no scale divisions, in difficult conditions, could have an uncertainty of 0.5 m and would be recorded as 6.5 ± 0.5 m.

It is useful to quote these uncertainties as percentages.

In the first 6.5 m the percentage uncertainty is $\frac{0.002}{6.500} \times 100\% = 0.03\%$. The measurement is 6.500 m ± 0.03%.

In the second 6.5 m the percentage uncertainty is $\frac{0.5}{6.5} \times 100\% = 7.69\%$. The measurement is 6.5 m ± 8%.

(Unless the percentage uncertainty is less than 1%, it is acceptable to quote percentage uncertainties to the nearest whole number.)

If the 6.5 m length is measured with a 5% error, the absolute error $= \frac{5}{100} \times 6.5\,\text{m} = \pm 0.325\,\text{m}$.

When a physical quantity is calculated, the uncertainty in the value is equal to the sum of all the percentage errors (not the sum of the absolute errors) in the quantities used in the calculation. The percentage uncertainty in the area of a rectangle with sides 5.6 ± 0.1 cm and 3.4 ± 0.1 cm

$\frac{0.1}{5.6} \times 100\% + \frac{0.1}{3.4} \times 100\% = 1.8\% + 2.9\% = 5\%$ (to nearest whole %).

PRACTICE QUESTIONS

1 Rewrite these measurements with the uncertainty shown as a percentage (to one significant figure):

a 5.7 ± 0.1 cm
b 2.0 ± 0.1 A
c 450 ± 2 kg
d 10.60 ± 0.05 s
e 47.5 ± 0.5 mV
f 366 000 ± 1000 J

2 Rewrite these measurements with the error shown as an absolute value:

a 1200 W ± 10%
b 34.1 m ± 1%
c 330 000 Ω ± 0.5%
d 0.00800 m ± 1%

3 Which of these measurements has the smallest percentage error?

A 9 ± 5 mm
B 26 ± 5 mm
C 516 ± 5 mm
D 1400 ± 5 mm

Significant figures

When you use a calculator to work out a numerical answer you know that this often results in a large number of decimal places and in most cases the final few digits are 'not significant.' The uncertainty in the data affects how many figures will be significant. It is important to record your data and your answers to calculations to a reasonable number of significant figures. Too many and your answer is claiming an accuracy that it does not have, too few and you are not showing the precision and care required in scientific analysis.

THE NUMBER OF SIGNIFICANT FIGURES

Three significant figures: 271 m 0.271 m 3.62 m 0.0345 m (notice that the zeros here just tell us how large the number is by showing where the decimal point goes, the three significant figures are underlined).

Three significant figures where the zero is significant:

207 m (any zero digits between the other significant digits will be significant).

27.0 m 0.350 m (in these cases extra decimal places are shown as zeros and this means these places are significant, 27 m and 0.35 m have only two significant figures).

Ambiguous significant figures:

270 m (2 or 3?) This is 2 s.f. unless it is written 270 m (3 s.f.) or 0.270 km or 2.70×10^2 m (see page 10).

35 000 kW (2 or more?) This is 2 s.f. unless it is written 35 000 kW (3 s.f.) or 35.0 MW or 3.50×10^4 W.

How many significant figures to use?

For practical data be guided by the uncertainty, as described on page 8.

For calculations, use the same number of figures as the data in the question with the lowest number of significant figures. It is not possible for the answer to be more accurate.

PRACTICE QUESTIONS

4 How many significant figures are there in these numbers?

a 609 W
b 3.4 kg
c 21.67 m
d 400.0 N
e 10.01 s
f 5 MW
g 6.0 s
h 9.8 m s^{-2}
i 3.0×10^8 m s^{-1}

5 Write these measurements to two significant figures:

a 19.47 m
b 115 km
c 21.0 s
d 6.63×10^{-34} J s
e 1.673×10^{-27} kg
f 5 s

6 Use the equation $V = IR$ to calculate the electric current I through a 3300 Ω resistance R when the voltage V = 12 V.

CALCULATIONS USING UNCERTAINTIES AND SIGNIFICANT FIGURES

In the example of the area of a rectangle on page 8, the area = 5.6 cm × 3.4 cm = 19.04 cm^2.

How many significant figures should we use?

The error was shown to be 5% so the absolute error = $\frac{5}{100} \times 19.04 = 0.952 = \pm 1$ cm^2.

So the '0' and the '4' are not significant, the answer is between 18 cm^2 and 20 cm^2. Two significant figures is appropriate and is the same number as in the two original length measurements. Area = 19 ± 1 cm^2.

PRACTICE QUESTIONS

7 A car travels 540 m in 16 s. Calculate the average speed.

8 Calculate the circumference and the area of a circular disc with radius 1.4 ± 0.1 cm. Give your answers with the uncertainty.