Developing maths skills for A level

As part of this online self-study course, you will be focusing on developing your maths skills for use with Physics A level, in areas such as: significant figures, standard form, units and prefixes, interpreting data from graphs and handling vectors.

Most of these skills you will already be familiar with, as you have been developing them throughout your GCSEs. It is helpful now to spend some time practising and improving these skills, to ensure a strong and confident start to your A level work in September.

Resources:

https://isaacphysics.org/
It will be helpful to make an account on Isaac Physics to help you track your process - and it will also allow you to join a class for this pre-A level course. Go to log in / sign up - just use KECHG for the school. You will need to click on an emailed link to activate your account. Then go to Menu / My Account / Teacher Connections and use the code 992HVJ to join our class. You will see that some assignments have already been set – just work through these as you progress through the course.

https://www.alevelphysicsonline.com/
Lots of useful resources here that you can use now and throughout your A level course. You can log in with the CHG account to access more resources. Use the email address physics@kechg.org.uk as user ID and the password *Ogden*.

Help and support:
If you have any questions about the work outlined here, please email r.nicholson@chg.kevibham.org for help and support.

Lesson 1: Units, dimensions, standard form and orders of magnitude

- Read the notes in the attached files *Units and dimensions.pdf* and *Scale of the Universe.pdf* and try the practice questions. This should be a re-cap of information that you already know but it is good to practice, as you will need to be confident working with very large and very small numbers at A level, using standard form and manipulating units. Check your answers using *answers-maths-skills-for-a-level-physics.pdf*.
- If you are not confident using standard form, take a look at these videos on the topic: https://www.youtube.com/playlist?list=PL3F89F1E677F1C1D0
- For more information on quantities, units and prefixes look at:
- https://www.alevelphysicsonline.com/quantities-and-units
- Isaac Physics for extra practice on standard form: https://isaacphysics.org/gameboards#phys19_a3

Lesson 2: Uncertainties and significant figures

- Read the file *Uncertainties and significant figures.pdf* and try the practice questions.
- For more practice on significant figures go to Isaac Physics: https://isaacphysics.org/questions/sig_fig_prac?board=sig_fig_prac_mastery (these questions have been set as an assignment).

- Watch the video on Accuracy, Precision, Error and Uncertainty at: https://www.alevelphysicsonline.com/practical-skills and write down definitions for these terms.
- On the same page, watch the videos on absolute uncertainty and percentage uncertainty and make sure you are clear on the difference. Try, for example, measuring the length and diameter with a 15cm ruler. The absolute uncertainty is the same for each value (1mm) but the percentage uncertainty is very different - why?
- Watch the video about finding the percentage uncertainty for multiple (repeated) measurements and write down a definition of how to do this. Try this out yourself - hold a sheet of A4 paper over your head and use your phone to time how long it takes for it to fall to the floor. Repeat 3 times and find the mean, the range and the uncertainty.

Lesson 3: Distance-time and speed-time graphs

Building on your GCSE knowledge, you will need to be able to understand and interpret distance-time and speed-time graphs, using the gradient of the distance-time graph to calculate speed, the gradient of the speed-time graph to calculate acceleration and the area under the speed-time graph to calculate distance travelled.

- Read through the notes, *Motion.pdf*, and have a go at all the practice questions.

Lesson 4: More practice with graphs

As part of A level physics, you will need to be able to interpret graphs, calculating gradients and areas under a graph and linking these to physical quantities.

- On Isaac Physics, try the following practice questions: https://isaacphysics.org/gameboards#phys19_a5
  It's worth having a go with these - one of the things that Isaac emphasises is thinking about units. Remember that a gradient always has a unit (which depends on the units on the y-axis and the units on the x-axis) and it is a good habit to get into to always think about what the units on a gradient will be.
- On Isaac Physics, try: https://isaacphysics.org/gameboards#phys19_a7 for practice on finding the area under a graph. Again, you will need to think about significant figures and units - for the units on an area you need to multiply the units on the x and y axes.

Lesson 5: Resultant forces

Building on our GCSE knowledge, the mechanics topic in A level looks in detail at static forces. You will need to be able to manipulate vectors: adding vectors to calculate resultant forces and also be able to resolving vectors into two perpendicular components. We'll start with this lesson, looking at resultant forces.

- Read the notes *Resultant forces.pdf* and have a go at the practice questions.
- If you have a printer, then print out the file Parallelogram of forces examples.pdf and have a go at the examples. There is one sheet with forces drawn on graph paper and to scale (make sure you print sized 100% so you don't change the scale) - and another version of this with completed answers so you can check your work.

- If you don't have a printer, you can have a go at drawing your own scale diagrams - shown in *force diagrams - video example sheet.pdf* - on plain or lined paper (you'll need a ruler and a protractor). The video here: https://www.youtube.com/watch?v=UQfNF-j8R7sFoc shows you how to do this step-by-step if you are not sure how.

Lesson 6: Resolving forces

- First, have a go at the examples in *Resolving Forces notes and q.doc*. There are some notes included to explain the theory, but you can also use the video here https://www.youtube.com/watch?v=rbtIiGSsE1A to help you work through the examples if you are not sure how to do this.
- Then, try the questions in *Resolving Forces Further Practice.doc* - you can check your answers with the mark scheme included in the document.
- Read the notes Resolving forces.pdf and have a go at the practice questions.

Units and dimensions

Base and derived SI units

Units are defined so that, for example, every scientist who measures a mass in kilograms uses the same size for the kilogram and gets the same value for the mass. Scientific measurement depends on standard units. Every measurement must give the unit to have any meaning. You should know the correct unit for physical quantities.

EXAMPLES OF QUANTITIES AND UNITS

Base units

Physical quantity	Unit	Symbol	Physical quantity	Unit	Symbol
length	metre	m	electric current	ampere	A
mass	kilogram	kg	temperature difference	kelvin	K
time	second	s	the amount of substance	mole	mol

Derived units

Example: Speed

Speed is defined as $\frac{\text{distance travelled}}{\text{time taken}}$.

If a car travels 2 metres in 2 seconds its speed is $\frac{2\text{ metres}}{2\text{ seconds}} = 1\frac{\text{m}}{\text{s}}$.

This defines the SI unit of speed to be 1 metre per second, which is written $1\,\text{m}\,\text{s}^{-1}$ $\left(\text{where } \text{s}^{-1} = \frac{1}{\text{s}}\right)$.

REMEMBER: Avoid unit errors: a numerical answer is incorrect if the unit is missing.

PRACTICE QUESTION

1 Complete this table with the missing units and symbols.

Physical quantity	Equation used to derive unit	Unit	Symbol and name (if there is one)
frequency	period^{-1}	s^{-1}	Hz hertz
volume	length^3		–
acceleration	velocity ÷ time		–
force	mass × acceleration	$\text{kg}\,\text{m}\,\text{s}^{-2}$	
work and energy	force × distance		J joule
voltage	energy ÷ electric charge	$\text{J}\,\text{C}^{-1}$	
electrical resistance		$\text{V}\,\text{A}^{-1}$	

Using base units to check equations and constants

The physical quantities that are measured in base units are called dimensions. Base units can be used to check if an equation is dimensionally correct. They can also be used to check the units of a constant.

CHECKING EQUATIONS

To check whether the equation Kinetic energy $= \frac{1}{2}mv^2$ is dimensionally correct.

Left-hand side of equation (LHS): $J = N\,m = kg\,m\,s^{-2} \times m = kg\,m^2\,s^{-2}$.

Right-hand side (RHS): $kg \times (m\,s^{-1})^2 = kg\,m^2\,s^{-2}$. (Notice that the constant $\frac{1}{2}$ has no units.)

The two sides are the same so the equation is dimensionally correct. We can't tell that there is $\frac{1}{2}$ in the equation so we can't tell that the equation is correct, only that it is dimensionally correct.

PRACTICE QUESTIONS

2 Use base units to show the equation $Q = It$ for electric charge passing a point in time t when the electric current is I, is dimensionally correct.

3 Use base units to show that the equation $P = IV$ is dimensionally correct, where I is electric current, V is voltage and P is power measured in watts (W) $1\,W = 1\,J\,s^{-1}$.

UNITS FOR CONSTANTS

The Planck constant h is used in the equation that tells us the energy of a photon: energy $E = hf$ where f is the frequency of the radiation. We can use this equation to find the units for h.

LHS: J

RHS: (units of h) s^{-1}

Units of h = J s

PRACTICE QUESTIONS

4 The Earth's gravitational field strength $g = 9.81\,N\,kg^{-1}$ is also sometimes given as the acceleration due to gravity $g = 9.81\,m\,s^{-2}$. Show that these units are equivalent.

5 Newton's law of gravitation says that the force F between two masses is $F = G\frac{mM}{r^2}$ where m and M are two masses and r is the distance between them. Find the units of the gravitational constant G.

REMEMBER: It's very important to use upper case letters and lower case letters correctly.
For example:
N for newton and m for metre.

STRETCH YOURSELF

Show that the formula for the period of a pendulum

$$T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$$ is dimensionally correct.

Scale of the Universe

Distant galaxies

When we describe the structure of the Universe we are using very large numbers. There are billions of galaxies and their average separation is about a million light years. The Big Bang theory says that the Universe began expanding about 14 billion years ago. The Sun formed about 5 billion years ago. These numbers and larger numbers can be expressed in standard form, and using prefixes.

USING STANDARD FORM

The diameter of the Earth is 13 000 km. 13 000 km = $1.3 \times 10\,000$ km = 1.3×10^4 km. In standard form the number is written with one digit in front of the decimal point and multiplied by the appropriate power of 10.

The distance to the Andromeda galaxy is 2 200 000 light years = $2.2 \times 1\,000\,000$ ly = 2.2×10^6 ly.

PRACTICE QUESTION

1 Write these measurements in standard form:

a 1350 W
b 503 N
c 130 000 Pa
d 86 400 s
e 696×10^6 s
f 9315×10^5 eV
g 0.176×10^{12} C kg^{-1}

ORDER OF MAGNITUDE CALCULATIONS

If a number is rounded to the nearest power of ten we say we are giving an order of magnitude value.

The average separation of the galaxies is $\sim 10^6$ light years. The symbol $\sim$ is used to mean 'to within an order of magnitude.'

The wavelength of red light is 700 nm and of violet light is 400 nm. They are both a few hundred nanometres so they are the same 'within an order of magnitude'.

PRACTICE QUESTIONS

2 Scientists estimate that the Big Bang occurred 13.7×10^9 years ago. Write this time as an order of magnitude.

3 Which planets are the same size, to within an order of magnitude? Radii: Mercury 2.4×10^6 m, Venus 6.09×10^6 m, Earth 6.4×10^6 m, Mars 3.4×10^6 m, Jupiter 7.1×10^7 m, Saturn 6.0×10^7 m, Uranus 2.4×10^7 m, Neptune 2.2×10^7 m.

PREFIXES

As an alternative to standard form, these prefixes are used with SI units. Drax power station has an output of 3.96×10^9 W. This can be written as 3960 MW or 3.96 GW.

Prefix	Symbol	Value	Prefix	Symbol	Value
kilo	k	10^3	giga	G	10^9
mega	M	10^6	tera	T	10^{12}

REMEMBER: Except for k, the symbols are all upper case. The factors increase in threes, that is 3, 6, 9, 12.

Particle theory

At the other end of the scale, the diameter of an atom is about a tenth of a billionth of a metre. The particles that make up an atomic nucleus are much smaller. These measurements are represented using negative powers of ten, and more prefixes.

POWERS OF TEN

One way to understand the negative powers of ten (or any number) is to write out a series and look at the pattern:

$1000 = 10^3, 100 = 10^2, 10 = 10^1, 1 = 10^0, 0.1 = \frac{1}{10} = 10^{-1}, 0.01 = \frac{1}{100} = 10^{-2}, 0.001 = \frac{1}{1000} = 10^{-3}$

To multiply powers of ten, add the indices: $1000 \times 100 = 100\,000$ becomes $10^3 \times 10^2 = 10^{(3+2)} = 10^5$.

To divide powers of ten, subtract the indices: $\frac{1000}{100} = 10$ becomes $\frac{10^3}{10^2} = 10^{(3-2)} = 10^1$.

To understand why $10^0 = 1$, think of $\frac{100}{100} = \frac{10^2}{10^2} = 10^{(2-2)} = 10^0 = 1$.

Dividing by 100 (or 10^2) is the same as multiplying by 0.01 (or 10^{-2}).

REMEMBER: $10^3 \times 10^{-2} = 10^{(3-2)} = 10^1 = 10$ but $10^3 + 10^{-2} = 1000.01$
You can only add and subtract the indices when you are multiplying or dividing the numbers, not adding or subtracting them.

PRACTICE QUESTION

4 The speed of light is $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$. Use the equation $v = f\lambda$ to calculate the frequency of:

a ultraviolet, wavelength 3.0×10^{-7} m
b radio waves, wavelength 1000 m
c X-rays, wavelength 1.0×10^{-10} m

SMALL NUMBERS: STANDARD FORM, ORDERS OF MAGNITUDE AND PREFIXES

In standard form the Planck constant $\lambda = 6.63 \times 10^{-34}$ J s.
The charge on an electron $= 1.6 \times 10^{-19}$ C.
As an order of magnitude, the diameter of an atom is $\sim 10^{-10}$ m and of a nucleus is $\sim 10^{-14}$ m.

Prefix	Symbol	Value	Prefix	Symbol	Value
centi	c	10^{-2}	nano	n	10^{-9}
milli	m	10^{-3}	pico	p	10^{-12}
micro	μ	10^{-6}	femto	f	10^{-15}

PRACTICE QUESTIONS

5 Write these measurements in standard form:

a 0.0025 m
b 0.60 kg
c 160×10^{-17} m
d 0.01×10^{-6} J
e 0.005×10^6 m
f 911×10^{-33} kg
g 0.00062×10^3 N

6 The charge on an electron is 1.6×10^{-19} C. Write this as an order of magnitude.

7 Write the measurements for question 1a, b, c, f, g on page 10 and question 5a, d, e above using suitable prefixes.

Uncertainties and significant figures

Uncertainties

When a physical quantity is measured there will always be a small difference between the measured value and the true value. How important the difference is depends on the size of the measurement and the size of the uncertainty, so it is important to know this information when using data. There are several possible reasons for uncertainty in measurements, including the difficulty of taking the measurement, the precision of the measuring instrument (for example due to the size of the scale divisions), and the natural variation of the quantity being measured. The word uncertainty is generally used in preference to error, because the word error is associated with something that is wrong. Mistakes in making measurements should be avoided, not included in the uncertainty.

EXAMPLES OF UNCERTAINTIES IN MEASUREMENTS

A measurement of 4.7 g on a scale with divisions of 0.1 g means the value is closer to 4.7 g than 4.6 g or 4.8 g. If the measurement was exactly half-way between 4.7 g and 4.8 g you would round up and record 4.8 g, so 4.7 g is anything from 4.65 g up to, but not including, 4.75 g and the measurement is written 4.7 ± 0.05 g.

A length of 6.5 m measured with great care and a 10 m tape measure marked in mm could have an uncertainty of 2 mm and would be recorded as 6.500 ± 0.002 m.

The same length measured with a stick 1 m in length and no scale divisions, in difficult conditions, could have an uncertainty of 0.5 m and would be recorded as 6.5 ± 0.5 m.

It is useful to quote these uncertainties as percentages.

In the first 6.5 m the percentage uncertainty is $\frac{0.002}{6.500} \times 100\% = 0.03\%$. The measurement is 6.500 m ± 0.03%.

In the second 6.5 m the percentage uncertainty is $\frac{0.5}{6.5} \times 100\% = 7.69\%$. The measurement is 6.5 m ± 8%.

(Unless the percentage uncertainty is less than 1%, it is acceptable to quote percentage uncertainties to the nearest whole number.)

If the 6.5 m length is measured with a 5% error, the absolute error $= \frac{5}{100} \times 6.5\text{ m} = \pm 0.325\text{ m}$.

When a physical quantity is calculated, the uncertainty in the value is equal to the sum of all the percentage errors (not the sum of the absolute errors) in the quantities used in the calculation. The percentage uncertainty in the area of a rectangle with sides 5.6 ± 0.1 cm and 3.4 ± 0.1 cm

$$\frac{0.1}{5.6} \times 100\% + \frac{0.1}{3.4} \times 100\% = 1.8\% + 2.9\% = 5\% \text{ (to nearest whole \%)}.$$

PRACTICE QUESTIONS

1 Rewrite these measurements with the uncertainty shown as a percentage (to one significant figure):

a 5.7 ± 0.1 cm
b 2.0 ± 0.1 A
c 450 ± 2 kg
d 10.60 ± 0.05 s
e 47.5 ± 0.5 mV
f 366 000 ± 1000 J

2 Rewrite these measurements with the error shown as an absolute value:

a 1200 W ± 10%
b 34.1 m ± 1%
c 330 000 Ω ± 0.5%
d 0.00800 m ± 1%

3 Which of these measurements has the smallest percentage error?

A 9 ± 5 mm
B 26 ± 5 mm
C 516 ± 5 mm
D 1400 ± 5 mm

Significant figures

When you use a calculator to work out a numerical answer you know that this often results in a large number of decimal places and in most cases the final few digits are 'not significant.' The uncertainty in the data affects how many figures will be significant. It is important to record your data and your answers to calculations to a reasonable number of significant figures. Too many and your answer is claiming an accuracy that it does not have, too few and you are not showing the precision and care required in scientific analysis.

THE NUMBER OF SIGNIFICANT FIGURES

Three significant figures: $\underline{271}$ m 0.$\underline{271}$ m $\underline{3.62}$ m 0.0$\underline{345}$ m (notice that the zeros here just tell us how large the number is by showing where the decimal point goes, the three significant figures are underlined).

Three significant figures where the zero is significant:

$\underline{207}$ m (any zero digits between the other significant digits will be significant).

$\underline{27.0}$ m 0.$\underline{350}$ m (in these cases extra decimal places are shown as zeros and this means these places are significant, 27 m and 0.35 m have only two significant figures).

Ambiguous significant figures:

270 m (2 or 3?) This is 2 s.f. unless it is written 270 m (3 s.f.) or 0.270 km or 2.70×10^2 m (see page 10).

35 000 kW (2 or more?) This is 2 s.f. unless it is written 35 000 kW (3 s.f.) or 35.0 MW or 3.50×10^4 W.

How many significant figures to use?

For practical data be guided by the uncertainty, as described on page 8.

For calculations, use the same number of figures as the data in the question with the lowest number of significant figures. It is not possible for the answer to be more accurate.

PRACTICE QUESTIONS

4 How many significant figures are there in these numbers?

a 609 W	b 3.4 kg	c 21.67 m
d 400.0 N	e 10.01 s	f 5 MW
g 6.0 s	h 9.8 m s^{-2}	i 3.0×10^8 m s^{-1}

5 Write these measurements to two significant figures:

a 19.47 m	b 115 km	c 21.0 s
d 6.63×10^{-34} J s	e 1.673×10^{-27} kg	f 5 s

6 Use the equation $V = IR$ to calculate the electric current I through a 3300 Ω resistance R when the voltage $V = 12$ V.

CALCULATIONS USING UNCERTAINTIES AND SIGNIFICANT FIGURES

In the example of the area of a rectangle on page 8, the area = 5.6 cm × 3.4 cm = 19.04 cm^2. How many significant figures should we use?

The error was shown to be 5% so the absolute error $= \frac{5}{100} \times 19.04 = 0.952 = \pm 1$ cm^2.

So the '0' and the '4' are not significant, the answer is between 18 cm^2 and 20 cm^2. Two significant figures is appropriate and is the same number as in the two original length measurements. Area = 19 ± 1 cm^2.

PRACTICE QUESTIONS

7 A car travels 540 m in 16 s. Calculate the average speed.

8 Calculate the circumference and the area of a circular disc with radius 1.4 ± 0.1 cm. Give your answers with the uncertainty.

Motion 1

Vectors and scalars

Scalar quantities have magnitude but no direction. Vector quantities have magnitude and direction.

WORKED EXAMPLE: SPEED, VELOCITY AND ACCELERATION

Speed is measured in metres per second, per means 'in each.' We can work out the average speed by measuring the distance travelled and dividing by the time taken. Speed is a scalar quantity this means it will always be positive. Distance and time are also scalars. They are always positive.

Velocity is a vector quantity. It is speed in a certain direction. When calculating velocity, v, we need to know the **displacement**, s. Displacement is the distance travelled in a certain direction. Displacement is a vector. For vectors, the opposite direction is the negative direction. Other directions are sometimes indicated by giving angles.

Acceleration is calculated from the change in velocity, Δv, divided by the time taken, Δt. (Δ is the symbol 'delta' and is used to mean 'a change in'.) Acceleration is the rate of change of velocity, not speed. This means that it is a vector. If an object is moving in a circle at constant speed, its direction is changing, so its velocity is changing, so it is accelerating.

PRACTICE QUESTIONS

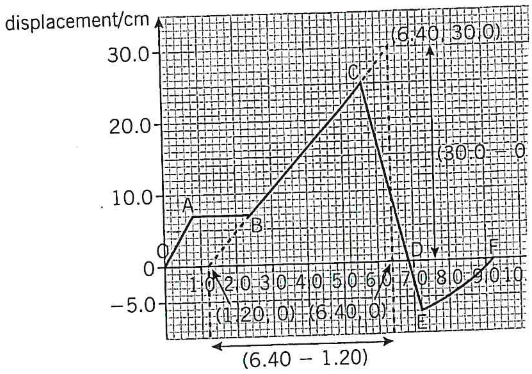
1. Divide these quantities into vectors and scalars: density, electric charge, electrical resistance, energy, field strength, force, friction, frequency, mass, momentum, power, voltage, volume, weight, work done.
2. Divide this data into vectors and scalars: $3\,m\,s^{-1}$, $+20\,m\,s^{-1}$, 100 m NE, 50 km, −5 cm, 10 km S 30° W.

STRETCH YOURSELF

Work done = force (N) × distance moved in direction of the force (m).
It is measured in joules, 1 J = 1 N m
A moment = force (N) × perpendicular distance from the pivot (m). It is measured in N m.
Explain whether these quantities and units are the same.

WORKED EXAMPLE: DISPLACEMENT-TIME GRAPHS

This graph shows the horizontal displacement of the cursor from a point on a monitor screen.



REMEMBER: Always read values to the nearest half, or quarter, of a small square – not to the nearest square.

Points to note are: OA constant velocity, AB stationary, BC constant but slower velocity, CE constant, faster, negative velocity. At D the cursor passes through the starting point and continues moving away from it, between E and F the velocity is positive and the cursor is accelerating towards the starting point – the displacement is negative until F when it is zero again.

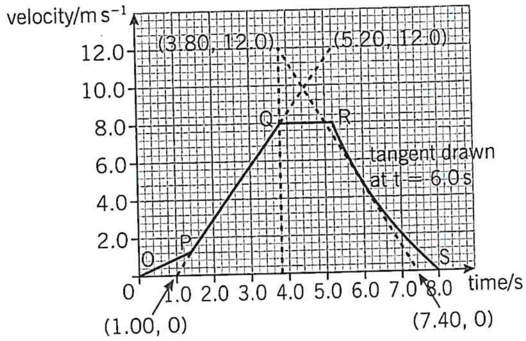
The velocity between B and C can be found from the gradient of the graph. To calculate the gradient, always draw the largest triangle possible to reduce uncertainties:

Gradient $= \dfrac{(30.0 - 0.0)\text{ cm}}{(6.40 - 1.20)\text{ s}} = 5.77\text{ cm s}^{-1}$. Don't forget the units, or to work out a reasonable number of significant figures based on how many you can read from the graph.

Velocity between BC $= 5.77\text{ cm s}^{-1}$ away from the point in the positive direction.

WORKED EXAMPLE: VELOCITY–TIME GRAPHS

This graph shows how the velocity of an object changes with time.



Points to note from this graph are: the object was stationary at O, accelerated with constant acceleration to P, PQ faster constant acceleration, QR constant velocity, RS slowing down, but not with uniform deceleration, stopping at S. The velocity was always positive so the object was moving in the same direction throughout.

The acceleration between P and Q can be found from the gradient of the graph.

Gradient $= \dfrac{(12.0 - 0.0)\text{ m s}^{-1}}{(5.20 - 1.00)\text{ s}} = 2.86\text{ m s}^{-2}$ acceleration $= 2.86\text{ m s}^{-2}$

The acceleration between R and S is negative. At any time it is equal to the gradient of the curve at that time. To find the acceleration at $t = 6.0$ s, we need to draw a tangent to the curve at the point where $t = 6.0$ s and find the gradient of the tangent. The tangent is the line that 'just touches' the curve at that point.

Gradient $= \dfrac{(0.0 - 12.0)\text{ m s}^{-1}}{(7.45 - 3.8)\text{ s}} = -3.29\text{ m s}^{-2}$ acceleration $= -3.29\text{ m s}^{-2}$

PRACTICE QUESTIONS

3 For the displacement–time graph on page 22, calculate the velocity between:

a O and A   b C and E.

4 For the velocity–time graph above, calculate the acceleration:

a between O and P   b at $t = 7.0$ s

Motion 2

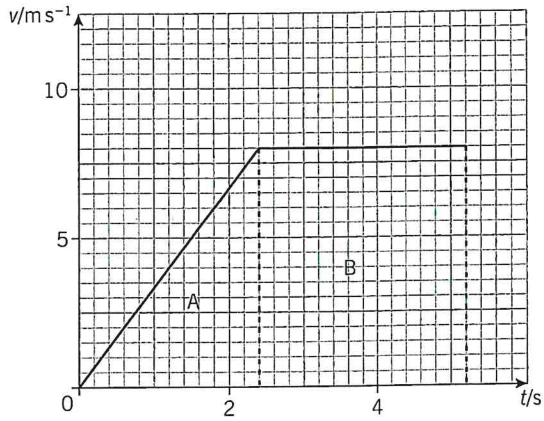
The distance travelled

The distance travelled by an object at constant velocity is equal to the velocity × time of travel. On a velocity–time graph, this is the area of the rectangle between the line and the time axis. For objects travelling with changing velocity, you can imagine that the area can be calculated from lots of very thin rectangles, with widths equal to very small time intervals, and height equal to the average velocity during that time interval.
This leads to the general conclusion that:

> the area under the curve of a velocity–time graph is equal to the distance travelled.

Areas can be calculated using the formulae for the area of a rectangle, a triangle or trapezium, or by 'counting the squares' under a curve.

WORKED EXAMPLE: TRIANGLES AND RECTANGLES



The area of the triangle marked A

$= \frac{1}{2}$ base × height

$= 0.5 \times 2.40\,\text{s} \times 8.0\,\text{m s}^{-1} = 9.6\,\text{m}^2$

The area of the rectangle marked B

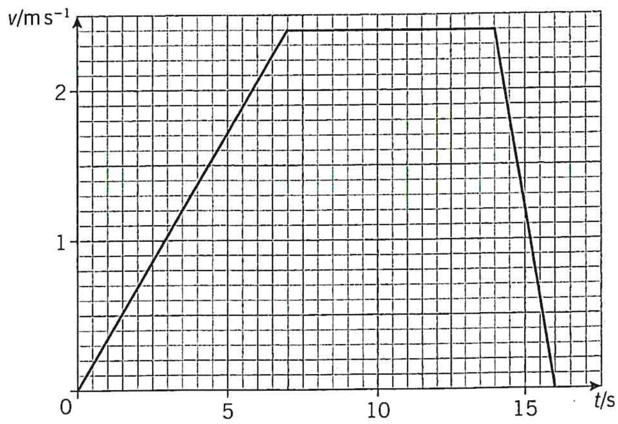
$=$ base × height

$= (5.20 - 2.40)\,\text{s} \times 8.0\,\text{m s}^{-1} = 22.4\,\text{m}^2$

The total distance travelled $= 9.6\,\text{m} + 22.4 = 32.0\,\text{m}$

$= 32\,\text{m}$ (2 s.f.)

WORKED EXAMPLE: TRAPEZIUM



The area of the trapezium

$= \frac{1}{2}$ (sum of parallel sides) × height

$= 0.5 \times (7.0 + 16.0)\,\text{s} \times 2.4\,\text{m s}^{-1}$

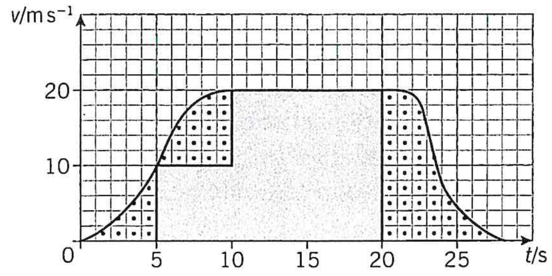
$= 27.6\,\text{m}^2$

The total distance travelled $= 28\,\text{m}$ (2 s.f.)

PRACTICE QUESTIONS

1. Use the formula for the area of a trapezium to check the area of the example above calculated by triangle and rectangle.
2. Use the triangle and rectangle formulae to check the area of the example above calculated by trapezium.

WORKED EXAMPLE: COUNTING SQUARES



Each small square has an area of $1.0\,s \times 2.0\,m\,s^{-1} = 2.0\,m^2$. A large square ($5 \times 5 = 25$) has an area of $25 \times 2.0\,m = 50\,m^2$. It is important to mark off the squares as you count them to avoid missing squares or double counting.

Count all the squares that are half or more under the curve, leave out those where less than half is under the curve.

Total squares = 5 large + 69 small

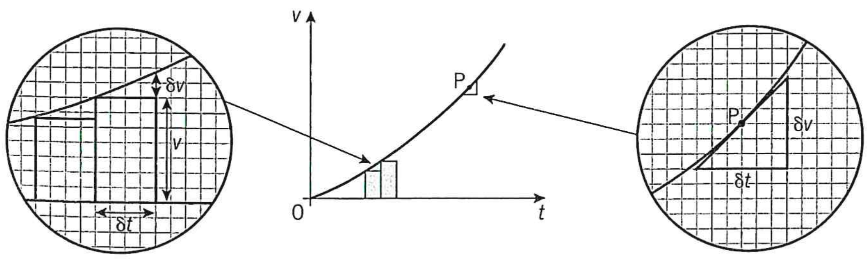
Distance travelled $\approx 5 \times 50\,m + 69 \times 2.0\,m = 250\,m + 138\,m$
$= 388\,m = 390\,m$ (3 s.f.)

PRACTICE QUESTION

3 Find the distance travelled for the velocity–time graph on page 23. (Hint: Use all three methods.)

More about curves

The area under a curve can be found by dividing it into very thin trapeziums of width δt, or sometimes Δt. (The symbol δ is also called 'delta', it means a very small change.) The total area is the sum of the areas of all the trapeziums. $\Sigma\,(v + \frac{1}{2}\,\delta v)\,\delta t$ where the symbol Σ means 'the sum of all'. As δt gets smaller and closer to zero, we write this $\delta t \to 0$, the value for the area gets closer to the true value.



The gradient of the curve at point P can be found by using a triangle where the base δt is very small. The gradient of the straight line at P is $\dfrac{\delta v}{\delta t}$.
As $\delta t \to 0$ the value for the gradient gets close to the true value.

It is then written $\dfrac{dv}{dt}$ and is called 'd v by d t' and is the gradient of a velocity–time (v–t) graph. It is the rate of change of v with t and is the acceleration at that point.

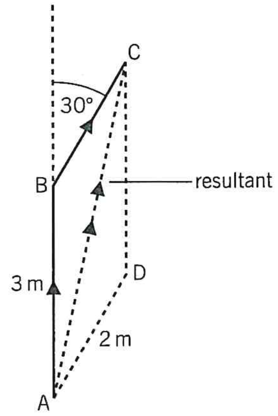
PRACTICE QUESTION

4 What is the value of $a = \dfrac{dv}{dt}$ at $t = 5.0\,s$ for the three graphs in the worked examples?

Forces

Resultant forces

Forces are vectors. When vectors are combined their direction must be taken into account. This diagram shows that walking 3 m from A to B and then turning through 30° and walking 2 m to C has the same effect as walking directly from A to C. $\overrightarrow{AC}$ is the **resultant** vector, denoted by the double arrowhead.

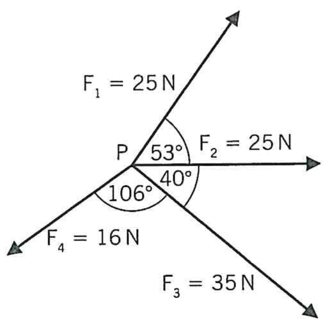


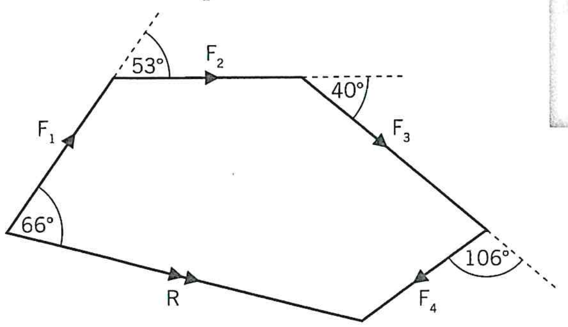
To combine forces, we can draw a similar diagram where the lengths of the sides represent the magnitude of the force (for example 30 N and 20 N). The third side of the triangle shows us the magnitude and direction of the resultant force. A careful drawing of a scale diagram allows us to measure these. Notice that if the vectors are combined by drawing them in the opposite order, $\overrightarrow{AD}$ and $\overrightarrow{DC}$, these are the other two sides of parallelogram and give the same resultant.

WORKED EXAMPLE: POLYGON OF FORCES

This result can be extended for many forces. Instead of a triangle we can draw a **polygon of forces**. This polygon is used to find the resultant of the four forces at P. Notice that each force starts from where the previous one ended, and the resultant is the direct route from the initial start point to the final end point.

> **REMEMBER:** If a set of forces is in equilibrium, then the last force in the polygon will join to the first force in the polygon and the resultant is zero.

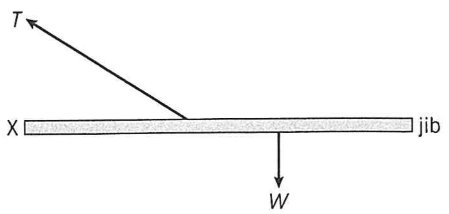




R = 53 N 66° clockwise from F_1

PRACTICE QUESTIONS

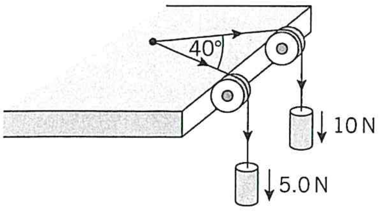
1 There are three forces on the jib of a tower crane. The tension in the cable T, the weight W, and a third force P acting at X.



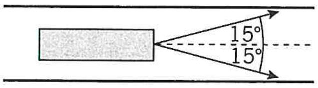
The crane is in equilibrium. Sketch the triangle of forces.

2 For each of these situations draw a triangle or polygon of forces to determine the resultant force:

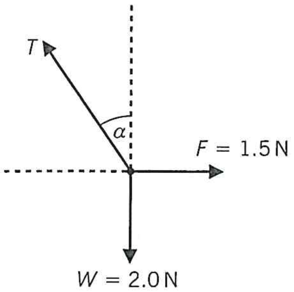
a



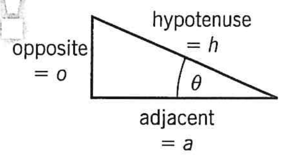
b Two forces of 5 kN towing a boat



3 These three forces are in equilibrium. Draw a triangle of forces to find T and α.



REMEMBER:



Pythagoras's theorem: For a right-angled triangle as shown: $h^2 = o^2 + a^2$

$\sin\theta = \frac{o}{h}$, $\cos\theta = \frac{a}{h}$

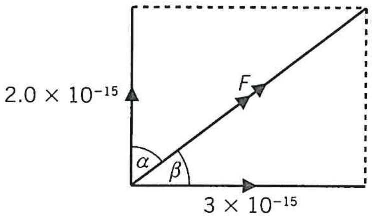
$\tan\theta = \frac{o}{a}$ (sohcahtoa)

Calculating resultants

When two forces are acting at right angles the resultant can be calculated using Pythagoras's theorem and trig functions, sine, cosine and tangent.

WORKED EXAMPLE

A sub-atomic particle experiences two forces at right angles, one of 2.0×10^{-15} N and the other 3.0×10^{-15} N.



$\alpha = \tan^{-1}\frac{3}{2} = 56°$

$\beta = \tan^{-1}\frac{2}{3} = 34°$

The resultant is represented by F.

$F^2 = (2.0 \times 10^{-15}\,\text{N})^2 + (3.0 \times 10^{-15}\,\text{N})^2$

$F = \sqrt{(4.0 + 9.0)} \times 10^{-15}\,\text{N}$

$F = 3.6 \times 10^{-15}\,\text{N}$

The angle is calculated using either $\tan\alpha$ or $\tan\beta$, remember to state, or show on the diagram, which angle you use. (This diagram shows both, but you only need to calculate one.)

PRACTICE QUESTION

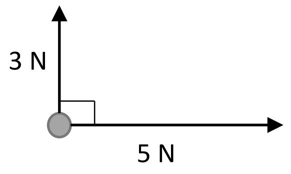
4 Find the resultant force for these pairs of forces at right angles:

a 3.0 N and 4.0 N

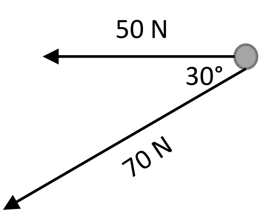
b 5.0 N and 12.0 N

Draw a scale diagram to find the resultant force in each of the following examples:

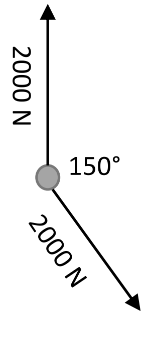
A



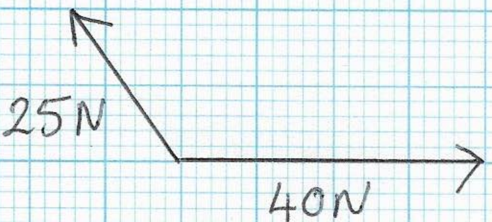
B



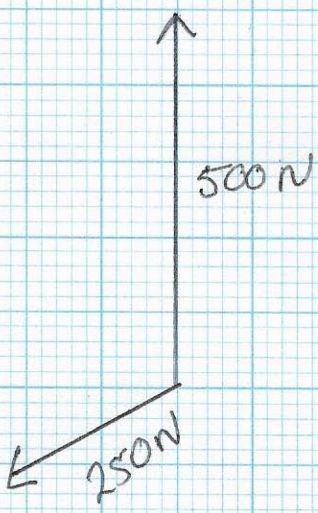
C

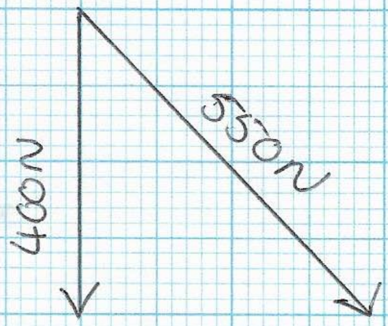


Scale 1 cm: 10 N

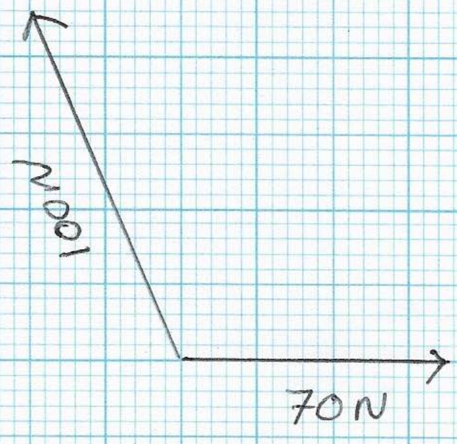


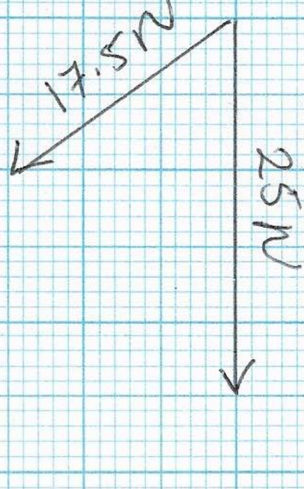
Scale 1 cm: 100 N



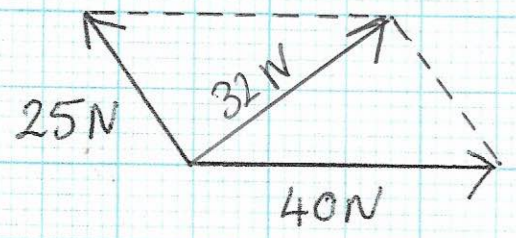


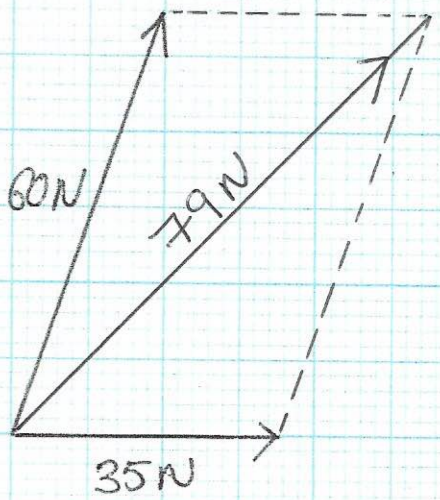
* What is the scale?
* What is the resultant force?





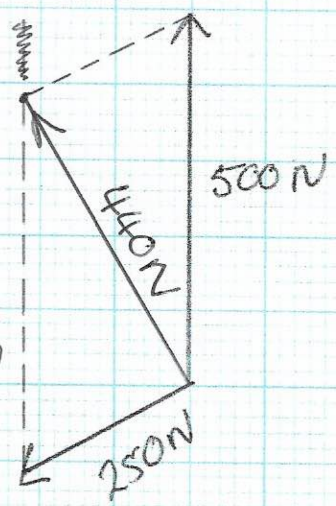
Scale 1cm: 10N

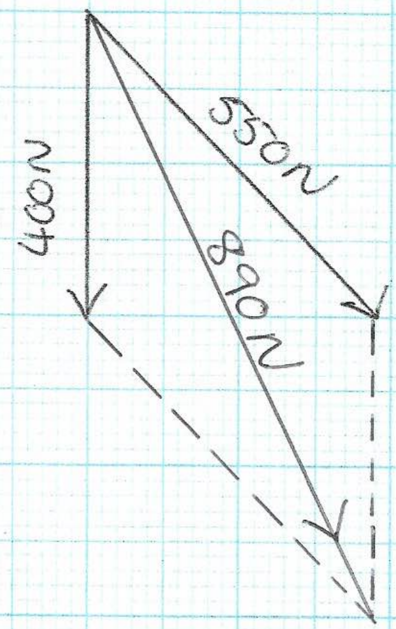




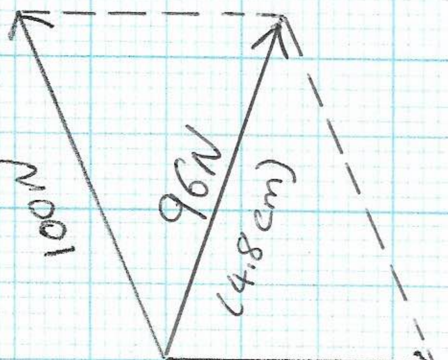
Scale 1cm: 100N

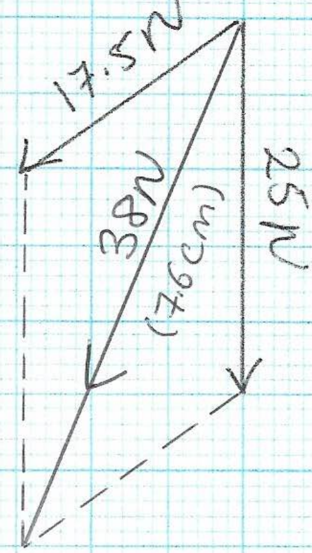
Make sure that you have:
- measured carefully
- included units
- considered the scale
- put an arrow on your force vectors





* What is the scale?
* What is the resultant force?

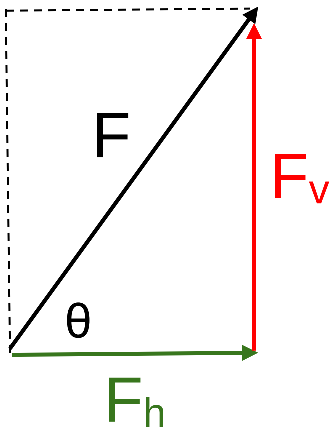




Scale 1cm: 20N

Scale 1cm: 5N

Resolving forces



As shown in the diagram, F is the resultant force of F_h and Fv. If these two forces act together on an object, one pulling to the right (F_h) and one pulling upward (F_v), then the resultant force, F is pulling up and to the right.

Resolving forces is like the reverse process of finding a resultant force - we take a force that's acting at an angle and work out how much force is pulling sideways and how much is pulling up or down.

Any diagonal force F can be resolved into two perpendicular forces - a horizontal component and a vertical component.

At GCSE, you can always resolve forces by using a scale diagram. Alternatively, you can use trigonometry to calculate the horizontal and vertical components.

Remember SOHCAHTOA and look at the triangle above.

$\sin\theta = F_v / F \quad \rightarrow \quad F_v = F\sin\theta$

$\cos\theta = F_h / F \quad \rightarrow \quad F_h = F\cos\theta$

(Take care with this: the horizontal component isn't always Fcosθ and the vertical component is not always Fsinθ. It depends on which angle you are given, so you will always need to draw a diagram and consider each situation individually.)

Practice:

Find the size of the horizontal and vertical components of the following forces:

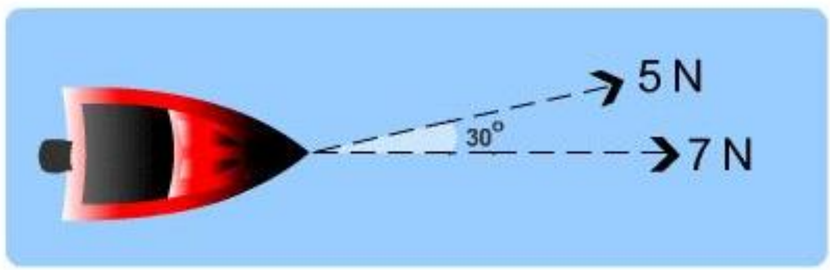
a) A force of 100N up and to the right, at 30° to the horizontal.
b) A 500N force down and to the left, at 40° to the vertical.
c) A 160N force up and to the left, at 25° to the horizontal.

<u>RESOLVING FORCES - PRACTICE QUESTIONS</u>

Remember to state the direction of the force.

1. Using a scale diagram AND trigonometry:

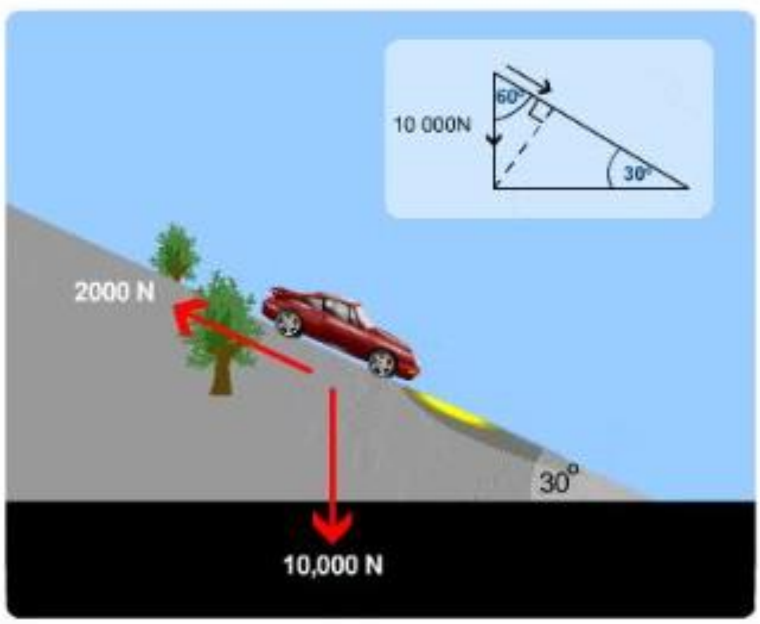
Find the forward force on the boat.



2. Using a scale diagram AND trigonometry:

Sometimes the direction we are interested in is not vertical or horizontal.
long as we still only add parallel forces.

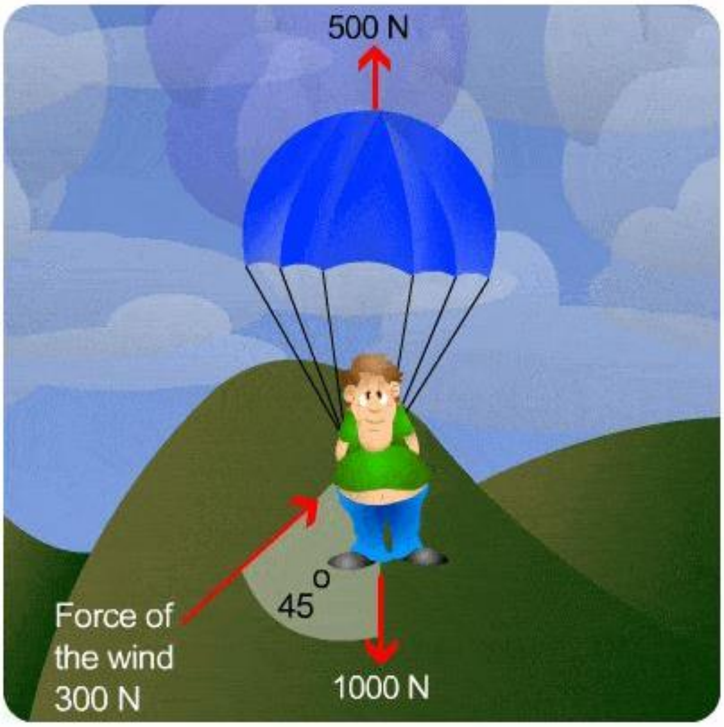
Find the force on the car parallel to the slope:



3. Using a scale diagram AND trigonometry:

(a) Find the horizontal force on the parachutist

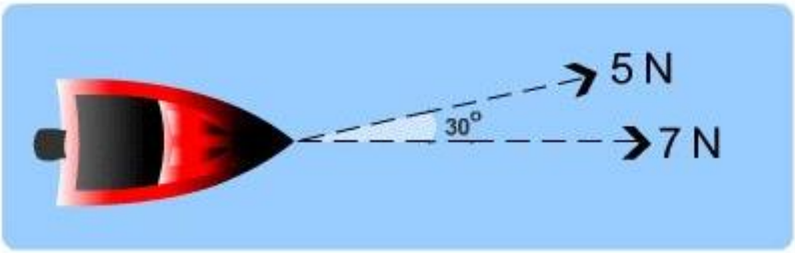
(b) Find the <u>overall</u> vertical force on the parachutist



Answers

1.

Find the forward force on the boat.



First you need to find the amount of the 5 N force that acts in the forward direction, using trigonometry:

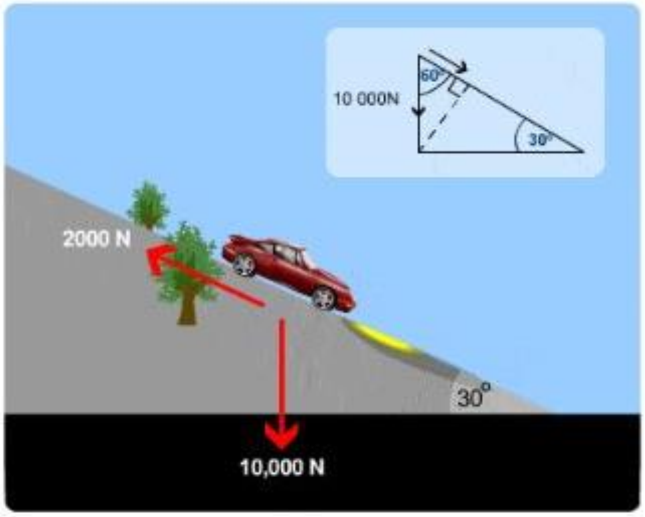
Part of 5 N force in forward direction = 5 cos 30° = 4.3 N

Then this can be added to the 7 N force:

4.3 + 7 = 11.3 N force in the forward direction.

2.

Find the force on the car parallel to the slope:



The 2000 N force is already parallel to the slope so we can ignore it for a moment.

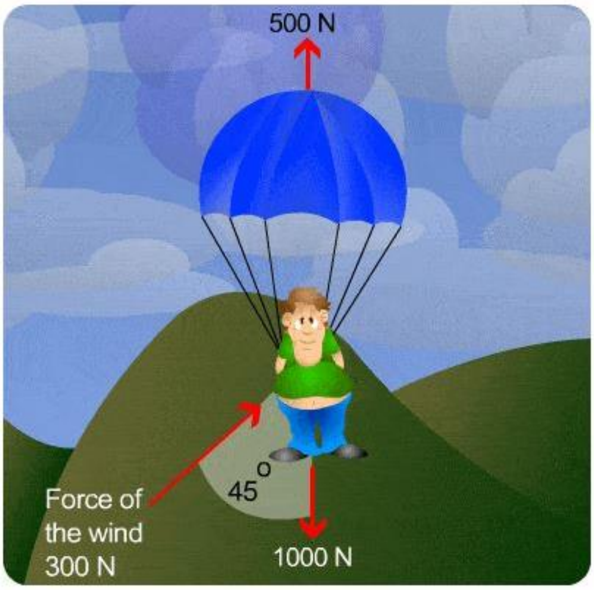
The 10 000 N is at an angle of 60 degrees to the slope so we need to use trigonometry to find its component parallel to the slope (look at the small triangle carefully):

Component parallel to the slope = 10 000 cos 60° = 5 000 N down the slope.

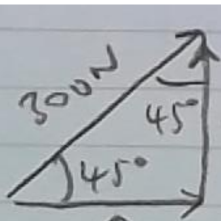
Now we can simply subtract the 2 000 N from the 5 000 N force as they are in opposite directions.

So the resultant force parallel to the slope = 5 000 - 2 000 = 3 000 N down the slope.

3.



3.



← 300 sin 45° (or 300 cos 45°) = 210 N

← 300 sin 45° (or 300 cos 45°) = 210 N

(a) HORIZONTAL FORCE = 210N TO THE RIGHT

(b) VERTICAL FORCE = 1000 − 500 − 210 = 290 N DOWNWARDS

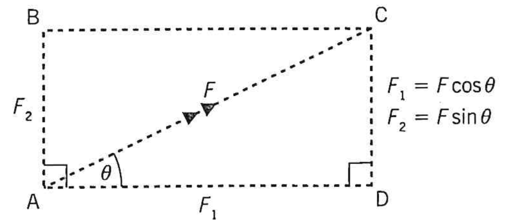
Resolving forces

Perpendicular forces

Just as we can find the **resultant** of several forces, we can replace a force with components that have the same effect, when combined, as the original force. This called **resolving** a force. Resolving a force into two components at right angles is the key to solving many problems.

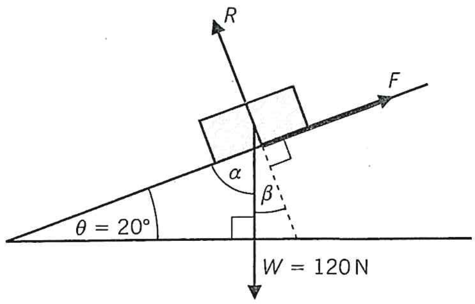
This diagram shows that when a force F is resolved into two components at right angles the components are:

$F_1 = F\cos\theta$ and $F_2 = F\sin\theta$ where θ is the angle between F and F_1.



WORKED EXAMPLE

A block of wood of weight, W, is placed on a slope. The friction, F, is large enough to prevent it slipping down.



REMEMBER: The angles of a triangle add up to 180°.
When two straight lines intersect, the opposite angles are always equal (because a straight line is 180°).
$\cos(90° - \theta) = \sin\theta$

The forces and motion parallel to the slope are independent from the forces and motion perpendicular to the slope. Resolving in these two directions simplifies the equations, as the friction has no effect perpendicular to the slope and the normal reaction R has no effect parallel to the slope.

The three forces are in equilibrium so:

Resolving perpendicular to the slope: $R = W\cos\beta = W\cos\theta$

$$R = (120\,\text{N})\cos 20° = 110\,\text{N}$$

Resolving parallel to the slope: $F = W\cos\alpha = W\sin\theta$

$$F = (120\,\text{N})\sin 20° = 41\,\text{N}$$

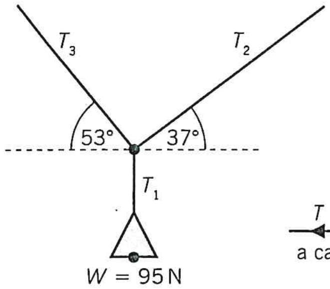
PRACTICE QUESTIONS

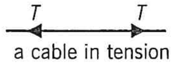
1 A force of 550 N is applied to a box at an angle of 30° to the horizontal. Calculate the horizontal and vertical components of the force.

2 Calculate the normal reaction and the friction for a box of weight 85 N in equilibrium on a slope of angle 15°.

WORKED EXAMPLE

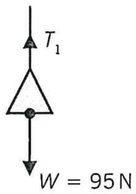
A lamp hangs from three cables tied as shown.

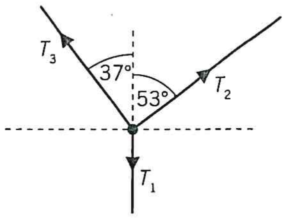




A string or cable can be in tension, but not in compression. If it is in tension, the force pulls on the objects it is attached to. The tension is the same at every point in the cable, or the cable would break, so it is always the same at both ends.

The lamp and the knot in the cable are both in equilibrium. Draw free body force diagrams for the lamp and the knot in the cable:





For the lamp: $T_1 = 95\text{ N}$ (1)
For the join in the cables: $\rightarrow$ (resolving horizontally)
$T_3 \cos 53° = T_2 \cos 37°$ (2)

$\uparrow$ (resolving vertically) $T_1 = T_3 \sin 53° + T_2 \sin 37°$ (3)

From (2): $T_3 = \dfrac{T_2 \cos 37°}{\cos 53°} = 1.33\ T_2$

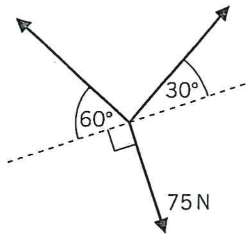
From (1) and (3) substitute for T_1 and T_3:
$95\text{ N} = 1.33\ T_2 \sin 53° + T_2 \sin 37°$
$95\text{N} = T_2\ (1.33 \sin 53° + \sin 37°) = T_2\ 1.66$

$T_2 = \dfrac{95\text{ N}}{1.66} = 57\text{ N}$

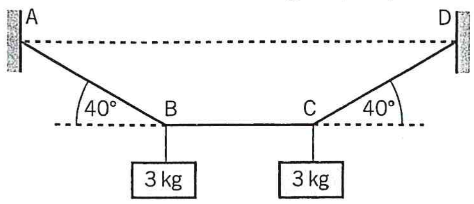
From (2) $T_3 = 1.33 \times 57\text{ N} = 76\text{ N}$

PRACTICE QUESTIONS

3 The three strings in the diagram are in tension and in equilibrium. Calculate the tension in each string.



4 Two masses are supported by three strings. BC is horizontal. What is the tension in string AB, BC, and CD?



5 A cable, parallel to a slope 30° to the horizontal, pulls a block up the slope at a steady speed. The block weighs 65 N and the friction with the slope is 12 N. What is the tension in the cable, and the normal reaction force?