Chapter 1: REMOVING BRACKETS

To remove a single bracket, we multiply every term in the bracket by the number or the expression on the outside:

Examples

1) $3(x + 2y) = 3x + 6y$

2) $-2(2x - 3) = (-2)(2x) + (-2)(-3)$

$= -4x + 6$

To expand two brackets, we must multiply everything in the first bracket by everything in the second bracket. We can do this in a variety of ways, including

* the smiley face method
* FOIL (Fronts Outers Inners Lasts)
* using a grid.

Examples:

1) $(x + 1)(x + 2) = x(x + 2) + 1(x + 2)$

or

$(x + 1)(x + 2) = x^2 + 2 + 2x + x$

$= x^2 + 3x + 2$

or

	x	1
x	x^2	x
2	$2x$	2

$(x + 1)(x + 2) = x^2 + 2x + x + 2$

$= x^2 + 3x + 2$

2) $(x - 2)(2x + 3) = x(2x + 3) - 2(2x + 3)$

$= 2x^2 + 3x - 4x - 6$

$= 2x^2 - x - 6$

or

$(x - 2)(2x + 3) = 2x^2 - 6 + 3x - 4x = 2x^2 - x - 6$

or

	x	-2
$2x$	$2x^2$	$-4x$
3	$3x$	-6

$(2x + 3)(x - 2) = 2x^2 + 3x - 4x - 6$

$= 2x^2 - x - 6$

EXERCISE A Multiply out the following brackets and simplify.

1. $7(4x + 5)$
2. $-3(5x - 7)$
3. $5a – 4(3a - 1)$
4. $4y + y(2 + 3y)$
5. $3x – (x + 4)$
6. $5(2x - 1) – (3x - 4)$
7. $(x + 2)(x + 3)$
8. $(t - 5)(t - 2)$
9. $(2x + 3y)(3x – 4y)$
10. $4(x - 2)(x + 3)$
11. $(2y - 1)(2y + 1)$
12. $(3 + 5x)(4 – x)$

Two Special Cases

Perfect Square:	**Difference of two squares:**
$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$	$(x - a)(x + a) = x^2 – a^2$
$(2x - 3)^2 = (2x – 3)(2x – 3) = 4x^2 – 12x + 9$	$(x - 3)(x + 3) = x^2 – 3^2$
	$= x^2 – 9$

EXERCISE B Multiply out

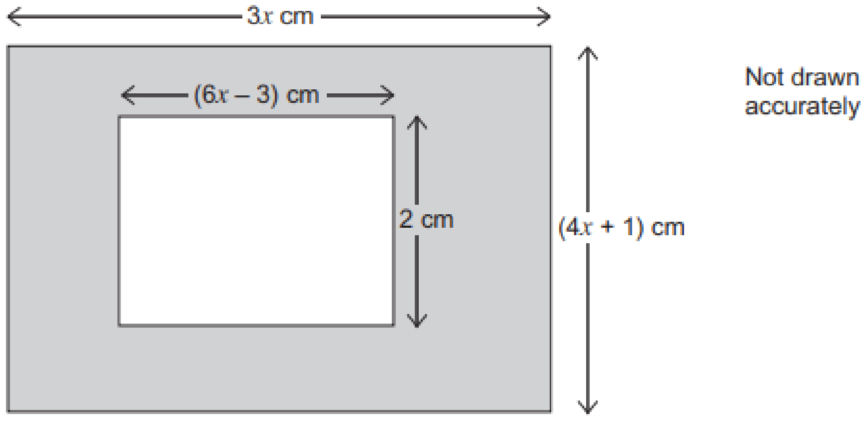
1. $(x - 1)^2$
2. $(3x + 5)^2$
3. $(7x - 2)^2$
4. $(x + 2)(x - 2)$
5. $(3x + 1)(3x - 1)$
6. $(5y - 3)(5y + 3)$

More help:
http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=brackets/bracketsMovie&taskID=1150

CHALLENGE QUESTIONS:

Question 1

The diagram shows two rectangles.



Show the shaded area, in cm ^{2}, is given by $12x^2 + bx + c$

....
(2 marks)

Question 2

$$3a(2x - 1) + 4(ax + 5) \equiv 60x + b$$

Work out the values of a and b.

....

Question 3

Simplify $(1 + x + y)^2 - (1 - x - y)^2$.

....

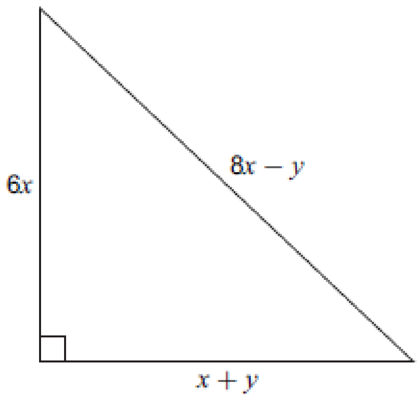
Question 4

The nth term of a sequence is $n^2 + 2n + 1$.
Find a fully simplified expression (in expanded form) for the $(n + 1)$th term of the sequence.

....

Question 5

The diagram shows a right-angled triangle.



Not drawn accurately

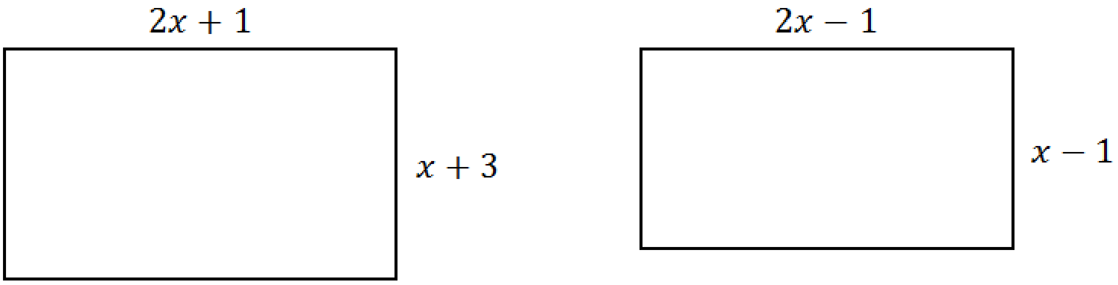
Find the ratio x: y, giving your answer in its simplest form.

.... : ....

(6 marks)

Question 6

The difference in area between the two rectangles below is 22. What is x?



$x =$ ....

Chapter 2: LINEAR EQUATIONS

When solving an equation, you must remember that whatever you do to one side must also be done to the other. You are therefore allowed to

- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, you should collect all the letters onto the same side of the equation.

If the equation contains brackets, you should start by expanding the brackets.

A linear equation is an equation that contains numbers and terms in x. A linear equation does not contain any x^2 or x^3 terms.

Example 1: Solve the equation $64 - 3x = 25$

Solution: There are various ways to solve this equation. One approach is as follows:

Step 1: Add $3x$ to both sides (so that the x term is positive): $64 = 3x + 25$

Step 2: Subtract 25 from both sides: $39 = 3x$

Step 3: Divide both sides by 3: $13 = x$

So the solution is $x = 13$.

Example 2: Solve the equation $6x + 7 = 5 - 2x$.

Solution:
Step 1: Begin by adding $2x$ to both sides
(to ensure that the x terms are together on the same side) $8x + 7 = 5$

Step 2: Subtract 7 from each side: $8x = -2$

Step 3: Divide each side by 8: $x = -¼$

Exercise A: Solve the following equations, showing each step in your working:

1) $2x + 5 = 19$ 2) $5x - 2 = 13$ 3) $11 - 4x = 5$

4) $5 - 7x = -9$ 5) $11 + 3x = 8 - 2x$ 6) $7x + 2 = 4x - 5$

Example 3: Solve the equation $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets: $6x - 4 = 20 - 3x - 6$
(taking care of the negative signs)

Step 2: Simplify the right hand side: $6x - 4 = 14 - 3x$

Step 3: Add 3x to each side: $9x - 4 = 14$

Step 4: Add 4: $9x = 18$

Step 5: Divide by 9: $x = 2$

Exercise B: Solve the following equations.

1) $5(2x - 4) = 4$

2) $4(2 - x) = 3(x - 9)$

3) $8 - (x + 3) = 4$

4) $14 - 3(2x + 3) = 2$

More help:
http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=equations/solvingEquationsMovie&taskID=1182

EQUATIONS CONTAINING FRACTIONS

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

Example 4: Solve the equation $\frac{y}{2} + 5 = 11$

Solution:
Step 1: Multiply through by 2 (the denominator in the fraction): $y + 10 = 22$

Step 2: Subtract 10: $y = 12$

Example 5: Solve the equation $\frac{1}{3}(2x + 1) = 5$

Solution:
Step 1: Multiply by 3 (to remove the fraction) $2x + 1 = 15$

Step 2: Subtract 1 from each side $2x = 14$

Step 3: Divide by 2 $x = 7$

When an equation contains two fractions, you need to multiply by the lowest common denominator.
This will then remove both fractions.

example 6: Solve the equation $\frac{x+1}{4}+\frac{x+2}{5}=2$

Solution:

Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator

$$\frac{20(x+1)}{4}+\frac{20(x+2)}{5}=40$$

Step 3: Simplify the left hand side:

$$\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}}+\frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}}=40$$

$5(x + 1) + 4(x + 2) = 40$

Step 4: Multiply out the brackets: $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation: $9x + 13 = 40$

Step 6: Subtract 13 $9x = 27$

Step 7: Divide by 9: $x = 3$

Example 7: Solve the equation $x+\frac{x-2}{4}=2-\frac{3-5x}{6}$

Solution: The lowest number that 4 and 6 go into is 12. So we multiply every term by 12:

$$12x+\frac{12(x-2)}{4}=24-\frac{12(3-5x)}{6}$$

Simplify	$12x+3(x-2)=24-2(3-5x)$
Expand brackets	$12x+3x-6=24-6+10x$
Simplify	$15x-6=18+10x$
Subtract $10x$	$5x-6=18$
Add 6	$5x = 24$
Divide by 5	$x = 4.8$

Exercise C: Solve these equations

1) $\frac{1}{2}(x+3)=5$

2) $\frac{2x}{3}-1=\frac{x}{3}+4$

3) $\frac{y}{4}+3=5-\frac{y}{3}$

4) $\frac{x-2}{7}=2+\frac{3-x}{14}$

5) $\frac{7x-1}{2}=13-x$

6) $\frac{y-1}{2}+\frac{y+1}{3}=\frac{2y+5}{6}$

7) $2x+\frac{x-1}{2}=\frac{5x+3}{3}$

8) $2-\frac{5}{x}=\frac{10}{x}-1$

More help:
http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=equations/solvingEquationsWithFractions&taskID=1183

FORMING EQUATIONS

Example 8: Find three consecutive numbers so that their sum is 96.

Solution: Let the first number be n, then the second is $n + 1$ and the third is $n + 2$.

Therefore $n + (n + 1) + (n + 2) = 96$

$3n + 3 = 96$

$3n = 93$

$n = 31$

So the numbers are 31, 32 and 33.

Exercise D:

1) Find 3 consecutive even numbers so that their sum is 108.

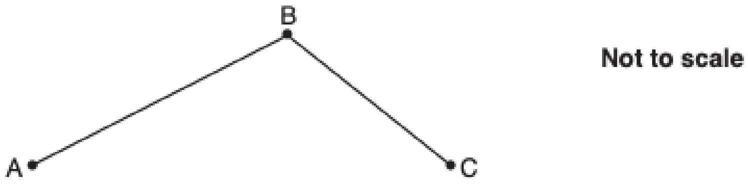
2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.

3) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number her friend has.
Form an equation, letting n be the number of photographs one girl had at the **beginning**.
Hence find how many each has **now**.

CHALLENGE QUESTIONS

Question 1

Halina cycled from A to B at an average speed of 26km per hour.
She then cycled from B to C at an average speed of 20km per hour.



She left A at 10.00am, did not stop at B and arrived at C at 3.00pm.
It took Halina x hours to cycle from A to B.
The distance from A to B is $26x$.
The distance from B to C is $100 - 20x$.
The **total distance** cycled by Halina from A to C is 118 km.
Find the distance from A to B.

……………………… km

(4 marks)

Question 2

Aaron, Bonny and Connor are playing a game with cards.
Aaron has some cards.
Bonny has twice as many cards as Aaron.
Connor has 6 cards more than Bonny.
They have a total of 101 cards.
Work out how many cards Aaron has.

……………………… cards

(4 marks)

Question 3

Given that $x + 1 : 4x = 2 : 7$, work out the value of x.

………………………

Question 4

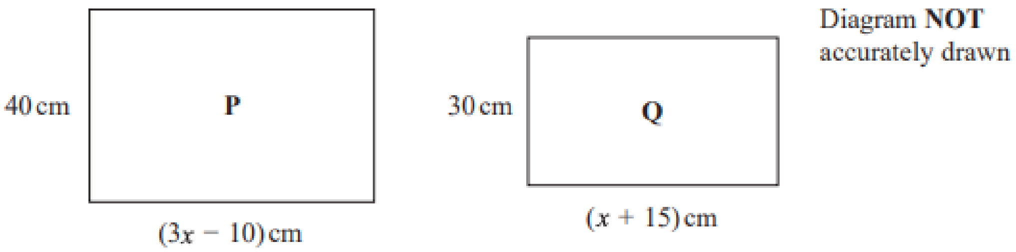
Solve

$$\frac{x}{4} - \frac{x+1}{2} + \frac{x+2}{3} - \frac{x+4}{6} = 2$$

………………………

Question 5

The diagram gives information about two paintings, P and Q.
Each painting is in the shape of a rectangle.



Painting P has an area 1400 cm^2 more than the area of painting Q.
Work out the area of painting P.

.... cm^2

(4 marks)

Question 6

Andrew divided some apples into six equal piles. Boris divided the same number of apples into five equal piles. Boris noticed that each of his piles contained two more apples than each of Andrew's piles.
How many apples did Andrew have?

.... apples

Chapter 3: SIMULTANEOUS EQUATIONS

An example of a pair of simultaneous equations is $3x + 2y = 8$ ①
$5x + y = 11$ ②

In these equations, *x* and *y* stand for two numbers. We can solve these equations in order to find the values of *x* and *y* by eliminating one of the letters from the equations.

In these equations it is simplest to eliminate *y*. We do this by making the coefficients of *y* the same in both equations. This can be achieved by multiplying equation ② by 2, so that both equations contain 2*y*:

$3x + 2y = 8$ ①
$10x + 2y = 22$ 2×② = ③

To eliminate the *y* terms, we subtract equation ③ from equation ①. We get: $7x = 14$
i.e. $x = 2$

To find y, we substitute $x = 2$ into one of the original equations. For example if we put it into ②:

$10 + y = 11$
$y = 1$

Therefore the solution is $x = 2$, $y = 1$.

Remember: You can check your solutions by substituting both x and y into the original equations.

Example: Solve $2x + 5y = 16$ ①
$3x - 4y = 1$ ②

Solution: We begin by getting the same number of *x* or *y* appearing in both equation. We can get 20*y* in both equations if we multiply the top equation by 4 and the bottom equation by 5:

$8x + 20y = 64$ ③
$15x - 20y = 5$ ④

As the SIGNS in front of 20*y* are DIFFERENT, we can eliminate the *y* terms from the equations by ADDING:

$23x = 69$ ③+④
i.e. $x = 3$

Substituting this into equation ① gives:

$6 + 5y = 16$
$5y = 10$
So… $y = 2$

The solution is $x = 3$, $y = 2$.

If you need **more help** on solving simultaneous equations, you can use the following website:
http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=simultaneous/simEquMovieHard&taskID=1174

Exercise:

Solve the pairs of simultaneous equations in the following questions:

1) $x + 2y = 7$
$3x + 2y = 9$

2) $x + 3y = 0$
$3x + 2y = -7$

3) $3x - 2y = 4$
$2x + 3y = -6$

4) $9x - 2y = 25$
$4x - 5y = 7$

5) $4a + 3b = 22$
$5a - 4b = 43$

6) $3p + 3q = 15$
$2p + 5q = 14$

CHALLENGE QUESTIONS:

Question 1

Karen has three times the number of cherries that Lionel has, and twice the number of cherries that Michael has. Michael has seven more cherries than Lionel. How many cherries do Karen, Lionel and Michael have altogether?

……………………… cherries

Question 2

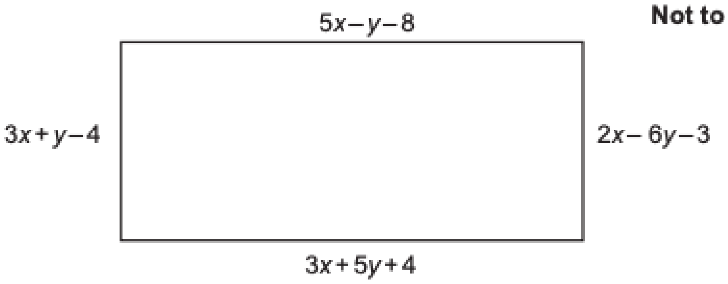
The sum of two positive integers is 97 and their difference is 37.
What is their product?

………………………

Question 3

The dimensions, in centimetres, of this rectangle are shown as algebraic expressions.

Not to scale



Work out the length and width of the rectangle.

length = …………………… cm

width = …………………… cm

(6 marks)

Question 4

Brachycephalus frogs are tiny - less than 1cm long - and have three toes on each foot and two fingers on each 'hand', whereas the common frog has five toes on each foot and four fingers on each 'hand'.

Some *Brachycephalus* and common frogs are in a bucket. Each frog has all its fingers and toes. Between them they have 122 toes and 92 fingers.

How many frogs are in the bucket?

…………………… frogs

Question 5

In the table shown, the sum of each row is shown to the right of the row and the sum of each column is shown below the column.

J	K	J	5
K	K	L	13
L	J	L	15
11	7	15	

What is the value of L?

L = ……………………

Chapter 4: FACTORISING

Common factors

We can factorise some expressions by taking out a common factor.

Example 1:	Factorise $12x - 30$
Solution:	6 is a common factor to both 12 and 30. We can therefore factorise by taking 6 outside a bracket: $12x - 30 = 6(2x - 5)$

Example 2:	Factorise $6x^2 - 2xy$
Solution:	2 is a common factor to both 6 and 2. Both terms also contain an x. So we factorise by taking $2x$ outside a bracket. $6x^2 - 2xy = 2x(3x - y)$

Example 3:	Factorise $9x^3y^2 - 18x^2y$
Solution:	9 is a common factor to both 9 and 18. The highest power of x that is present in both expressions is x^2. There is also a y present in both parts. So we factorise by taking $9x^2y$ outside a bracket: $9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$

Example 4:	Factorise $3x(2x - 1) - 4(2x - 1)$
Solution:	There is a common bracket as a factor. So we factorise by taking $(2x - 1)$ out as a factor. The expression factorises to $(2x - 1)(3x - 4)$

Exercise A

Factorise each of the following

1) $3x + xy$

2) $4x^2 - 2xy$

3) $pq^2 - p^2q$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y - 1) + 3(y - 1)$

Factorising quadratics

Simple quadratics: Factorising quadratics of the form $x^2 + bx + c$
The method is:
Step 1: Form two brackets $(x \ \ldots \ )(x \ \ldots \ )$
Step 2: Find two numbers that multiply to give c and add to make b. These two numbers get written at the other end of the brackets.

Example 1: Factorise $x^2 - 9x - 10$.

Solution: We need to find two numbers that multiply to make -10 and add to make -9.
These numbers are -10 and 1.
Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

General quadratics: Factorising quadratics of the form $ax^2 + bx + c$
The method is:
Step 1: Find two numbers that multiply together to make ac and add to make b.
Step 2: Split up the bx term using the numbers found in step 1.
Step 3: Factorise the front and back pair of expressions as fully as possible.
Step 4: There should be a common bracket. Take this out as a common factor.

Example 2: Factorise $6x^2 + x - 12$.

Solution: We need to find two numbers that multiply to make $6 \times -12 = -72$ and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = \underbrace{6x^2 - 8x}\ \underbrace{+ 9x - 12}$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

Difference of two squares: Factorising quadratics of the form $x^2 - a^2$

Remember that $x^2 - a^2 = (x + a)(x - a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$
$16x^2 - 25 = (2x)^2 - 5^2 = (2x + 5)(2x - 5)$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x + 4)(x - 4)$
and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x + 4y)(x - 4y)$

Factorising by pairing
We can factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing:

$2x^2 + xy - 2x - y = x(2x + y) - 1(2x + y)$ (factorise front and back pairs, ensuring both brackets are identical)

$= (2x + y)(x - 1)$

If you need **more help** with factorising, you can use this website:

http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=factorising/factoriseHigher&taskID=1156

Exercise B

Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$ (factorise by taking out a common factor)

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

13) $4y^3 - 9a^2y$

14) $8(x+1)^2 - 2(x+1) - 10$

CHALLENGE QUESTIONS

Question 1

Factorise the following fully: 3a(x-2)+6c(2x-4)

....

Question 2

Simplify $$\frac{(a+c)(x+1)+3x+3}{2bx+2b}$$

....

Question 3

Simplify

$$\frac{x^2 - 8x + 12}{x^2 - 7x + 6}$$

....

Question 4

The nth term of a sequence is $n^2 + 12n + 27$
By factorising, or otherwise, show that the 20th term can be written as the product of two prime numbers.

....
(2 marks)

Question 5

Fully factorise $(3x^2 - x - 6)^2 - (2x^2 - x + 3)^2$

....

Question 6

Show that

$$\frac{1}{2x^2 + x - 15} \div \frac{1}{3x^2 + 9x}$$

simplifies to $\frac{ax}{bx+c}$, where a, b and c are integers to be found.

....
(3 marks)

Question 7

It can be shown that $a^2 - b^2 \equiv (a-b)(a+b)$
Hence, or otherwise, simplify fully $(x^2 + 4)^2 - (x^2 - 2)^2$

....
(3 marks)

Question 8

The teenagers Sam and Jo notice the following facts about their ages:
The difference between the squares of their ages is four times the sum of their ages.
The sum of their ages is eight times the difference between their ages.
What is the age of the older of the two?

.... year-old

Chapter 5: CHANGING THE SUBJECT OF A FORMULA

We can use algebra to change the subject of a formula. Rearranging a formula is similar to solving an equation – we must do the same to both sides in order to keep the equation balanced.

Example 1: Make x the subject of the formula $y = 4x + 3$.

Solution: $y = 4x + 3$

Subtract 3 from both sides: $y - 3 = 4x$

Divide both sides by 4; $\frac{y-3}{4} = x$

So $x = \frac{y-3}{4}$ is the same equation but with x the subject.

Example 2: Make x the subject of $y = 2 - 5x$

Solution: Notice that in this formula the x term is negative.

$y = 2 - 5x$

Add $5x$ to both sides $y + 5x = 2$ (the x term is now positive)

Subtract y from both sides $5x = 2 - y$

Divide both sides by 5 $x = \frac{2-y}{5}$

Example 3: The formula $C = \frac{5(F-32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.

We can rearrange to make F the subject.

$C = \frac{5(F-32)}{9}$

Multiply by 9 $9C = 5(F-32)$ (this removes the fraction)

Expand the brackets $9C = 5F - 160$

Add 160 to both sides $9C + 160 = 5F$

Divide both sides by 5 $\frac{9C+160}{5} = F$

Therefore the required rearrangement is $F = \frac{9C+160}{5}$.

Exercise A

Make x the subject of each of these formulae:

1) $y = 7x - 1$

2) $y = \frac{x+5}{4}$

3) $4y = \frac{x}{3} - 2$

4) $y = \frac{4(3x-5)}{9}$

Rearranging equations involving squares and square roots

Example 4: Make x the subject of $x^2 + y^2 = w^2$

Solution: $x^2 + y^2 = w^2$

Subtract y^2 from both sides: $x^2 = w^2 - y^2$ (this isolates the term involving x)

Square root both sides: $x = \pm\sqrt{w^2 - y^2}$

Remember that you can have a positive or a negative square root. We cannot simplify the answer any more.

Example 5: Make a the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Solution: $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Multiply by 4: $4t = \sqrt{\frac{5a}{h}}$

Square both sides: $16t^2 = \frac{5a}{h}$

Multiply by h: $16t^2 h = 5a$

Divide by 5: $\frac{16t^2 h}{5} = a$

Exercise B:

Make t the subject of each of the following

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

More help:
http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=simplify/rearrangehigher&taskID=1170

More difficult examples

Sometimes the variable that we wish to make the subject occurs in more than one place in the formula. In these questions, we collect the terms involving this variable on one side of the equation, and we put the other terms on the opposite side.

Example 6: Make *t* the subject of the formula $a - xt = b + yt$

Solution: $a - xt = b + yt$

Start by collecting all the t terms on the right hand side:

Add *xt* to both sides: $a = b + yt + xt$

Now put the terms without a *t* on the left hand side:

Subtract *b* from both sides: $a - b = yt + xt$

Factorise the RHS: $a - b = t(y + x)$

Divide by (*y* + *x*): $\frac{a-b}{y+x} = t$

So the required equation is $t = \frac{a-b}{y+x}$

Example 7: Make *W* the subject of the formula $T - W = \frac{Wa}{2b}$

Solution: This formula is complicated by the fractional term. We begin by removing the fraction:

Multiply by 2*b*: $2bT - 2bW = Wa$

Add 2*bW* to both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by *a* + 2*b*: $W = \frac{2bT}{a+2b}$

Exercise C

Make *x* the subject of these formulae:

1) $ax + 3 = bx + c$

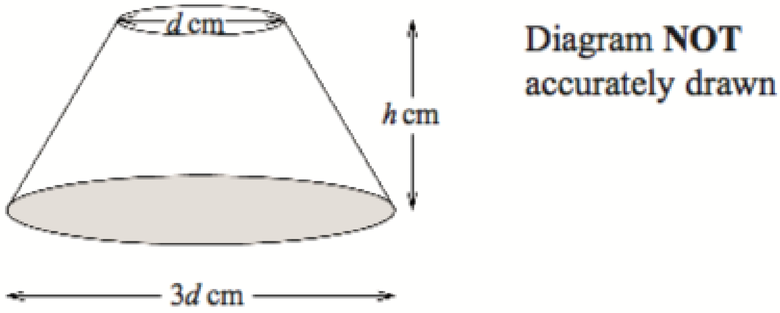
2) $3(x + a) = k(x - 2)$

3) $y = \frac{2x+3}{5x-2}$

4) $\frac{x}{a} = 1 + \frac{x}{b}$

CHALLENGE QUESTIONS:

Question 1



The diagram shows a frustum.
The diameter of the base is $3d$ cm and the diameter of the top is d cm. The height of the frustum is h cm.
The formula for the curved surface area, Scm^2, of the frustum is

$$S = 2\pi d\sqrt{h^2 + d^2}$$

Rearrange the formula to make h the subject.

$h =$ ....
(3 marks)

Question 2

$$y = at^2 - 2at$$
$$x = 2a\sqrt{t}$$

Express $y$in terms of $x$and a.
Give your answer in the form

$$y = \frac{x^p}{ma^3} - \frac{x^q}{na}$$

where p, q, m and $n$are integers.

$y =$ ....
(4 marks)

Question 3

Make q the subject of

$$p = \frac{1}{1 + \frac{1}{1 + \frac{1}{q}}}$$

$q =$ ....

Question 4

Make $x$the subject of the equation, giving your answer **as a single fraction** (and no fractions within fractions):

$$\frac{1}{1 - \sqrt{y}} = 1 - \frac{1}{x}$$

$x =$ ……………………

Question 5

Make $x$the subject of the following, **fully simplifying your answer**.

$$y + 2 = \frac{x + 1}{x - y}$$

(Note that the left-hand-side of the equation is already written for you)

$x =$ ……………………

Question 6

Make $y$the subject of

$$\frac{y}{x} + \frac{2y}{x + 4} = 3$$

Give your answer as simply as possible.

$y =$ ……………………

(5 marks)

Chapter 6: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations:
* factorising
* the quadratic formula

Note that not all quadratic equations can be solved by factorising. The quadratic formula can always be used however.

Method 1: Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of x^2 is positive.

Example 1 : Solve $x^2 - 3x + 2 = 0$

Factorise $(x - 1)(x - 2) = 0$
Either $(x - 1) = 0$ or $(x - 2) = 0$
So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

Example 2: Solve $x^2 - 2x = 0$

Factorise: $x(x - 2) = 0$
Either $x = 0$ or $(x - 2) = 0$
So $x = 0$ or $x = 2$

http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=factorising/solveQuadsByFactoring&taskID=1181

Method 2: Using the formula

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3: Solve the equation $2x^2 - 5 = 7 - 3x$

Solution: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$
We can then tell that $a = 2$, $b = 3$ and $c = -12$.
Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the *surd form* for the solutions)

If we have a calculator, we can evaluate these roots to get: $x = 1.81$ or $x = -3.31$

If you need more help with the work in this chapter, you can get help from this web site:

http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=quadraticformula/formulamovie&taskID=1160

EXERCISE

1) Use factorisation to solve the following equations:

a) $x^2 + 3x + 2 = 0$

b) $x^2 - 3x - 4 = 0$

c) $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a) $x^2 + 3x = 0$

b) $x^2 - 4x = 0$

c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a) $6x^2 - 5x - 4 = 0$

b) $8x^2 - 24x + 10 = 0$

4) Use the formula to solve the following equations to 3 significant figures. Some of the equations can't be solved.

a) $x^2 + 7x + 9 = 0$

b) $6 + 3x = 8x^2$

c) $4x^2 - x - 7 = 0$

d) $x^2 - 3x + 18 = 0$

e) $3x^2 + 4x + 4 = 0$

f) $3x^2 = 13x - 16$

CHALLENGE QUESTIONS:

Question 1

I am thinking of a number.

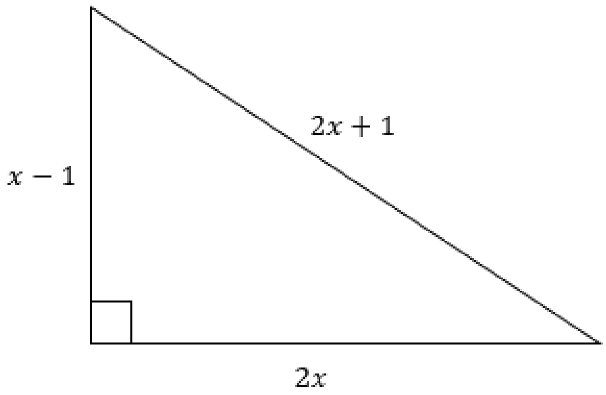
> When I subtract 25 from my number, then square the answer,
>
> I get the **same result as**
>
> when I square my number, then subtract 25 from the answer.

What is my number?

my number is ....

Question 2

By first forming an appropriate equation, determine the value of x.



$x =$ ....

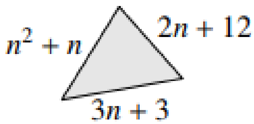
Question 3

The product of two positive integers is equal to twice their sum. The product is also equal to six times the difference between the two integers. What is the sum of these two integers?

....

Question 4

The diagram shows a triangle with sides n^2+n, $2n+12$ and $3n+3$.



What is the sum of all the values of n for which the triangle is isosceles?

....

Question 5

A designer is lining the base and sides of a rectangular drawer with paper.
The width of the drawer is $2x$ cm, the length is $3x$ cm and the height is 5 cm.
The drawer has no top. The total area of paper is 4070 cm^2.

Find the value of x to 1 decimal place, and use this value of x to work out the volume $V$of the drawer in litres, giving your answer to 1 decimal place.

$x =$ .... cm

$V =$ .... litres

Question 6

Alison is using the quadratic formula to solve a quadratic equation.
She substitutes values into the formula and correctly gets

$$x = \frac{-7 \pm \sqrt{49 - 32}}{4}$$

Work out the quadratic equation that Alison is solving.
Give your answer in the form $ax^2 + bx + c = 0$, where a, $b$and $c$are integers.

....

(3 marks)

Question 7

Solve the equation

$$\frac{1}{x-2} - \frac{1}{x-1} = 2$$

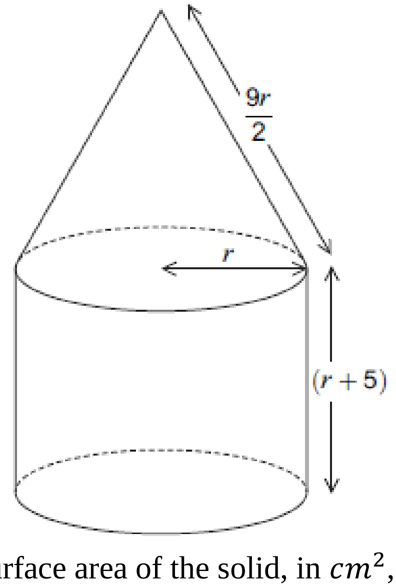
Give your answers to 2 decimal places.

....

(6 marks)

Question 8

On this diagram all lengths are given in centimetres.
A cylinder and cone are joined together to make a solid.
The cylinder has radius r and height $r + 5$
The cone has radius r and slant height $\frac{9r}{2}$



It can be shown that the total surface area of the solid, in cm^2, is $\frac{5\pi r}{2}(3r + 4)$
The total surface area of the solid is $1200\ picm^{\ 2}$
Work out the value of r.

....

Chapter 7: INDICES

Basic rules of indices

y^4 means $y \times y \times y \times y$.

4 is called the **index** (plural: indices), **power** or **exponent** of *y*.

There are 3 basic rules of indices:

1) $a^m \times a^n = a^{m+n}$ e.g. $3^4 \times 3^5 = 3^9$
2) $a^m \div a^n = a^{m-n}$ e.g. $3^8 \times 3^6 = 3^2$
3) $(a^m)^n = a^{mn}$ e.g. $\left(3^2\right)^5 = 3^{10}$

Further examples

$y^4 \times 5y^3 = 5y^7$

$4a^3 \times 6a^2 = 24a^5$ (multiply the numbers and multiply the *a*'s)

$2c^2 \times \left(-3c^6\right) = -6c^8$ (multiply the numbers and multiply the *c*'s)

$24d^7 \div 3d^2 = \dfrac{24d^7}{3d^2} = 8d^5$ (divide the numbers and divide the *d* terms i.e. by subtracting the powers)

Exercise A

Simplify the following:

1) $b \times 5b^5 =$ (Remember that $b = b^1$)

2) $3c^2 \times 2c^5 =$

3) $b^2c \times bc^3 =$

4) $2n^6 \times (-6n^2) =$

5) $8n^8 \div 2n^3 =$

6) $d^{11} \div d^9 =$

7) $\left(a^3\right)^2 =$

8) $\left(-d^4\right)^3 =$

Help:
http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=powers/indicesPart2&taskID=1045

More complex powers

Zero index:
Recall from GCSE that

$$a^0 = 1.$$

This result is true for any non-zero number *a*.

Therefore $5^0 = 1$ $\left(\frac{3}{4}\right)^0 = 1$ $(-5.2304)^0 = 1$

Negative powers

A power of -1 corresponds to the reciprocal of a number, i.e. $a^{-1} = \frac{1}{a}$

Therefore $5^{-1} = \frac{1}{5}$

$$0.25^{-1} = \frac{1}{0.25} = 4$$

$$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$$

(you find the reciprocal of a fraction by swapping the top and bottom over)

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

This means:

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$$

Fractional powers:
Fractional powers correspond to roots:

$a^{1/2} = \sqrt{a}$ $a^{1/3} = \sqrt[3]{a}$ $a^{1/4} = \sqrt[4]{a}$

In general:

$$a^{1/n} = \sqrt[n]{a}$$

Therefore:

$8^{1/3} = \sqrt[3]{8} = 2$ $25^{1/2} = \sqrt{25} = 5$ $10000^{1/4} = \sqrt[4]{10000} = 10$

A more general fractional power can be dealt with in the following way: $a^{m/n} = \left(a^{1/n}\right)^m$

So $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

$$\left(\frac{8}{27}\right)^{2/3} = \left(\left(\frac{8}{27}\right)^{1/3}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{25}{36}\right)^{-3/2} = \left(\frac{36}{25}\right)^{3/2} = \left(\sqrt{\frac{36}{25}}\right)^{3} = \left(\frac{6}{5}\right)^{3} = \frac{216}{125}$$

Help:
http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=powers/indicesPart3&taskID=1301

CHALLENGE QUESTIONS

Question 1

Simplify $8 \times 4^3 + 2 \times 4^4$ giving your answer in the form 4^b

....
(4 marks)

Question 2

Solve the equation

$$9^{(2x^2+2x)} = 27$$

....

Question 3

Suppose that $8^m = 27$.
What is the value of 4^m?

....

Question 4

Given that $3^{-n} = 0.2$, find the value of $(3^4)^n$

....
(2 marks)

Question 5

$x^{\frac{1}{2}} = 6$ and $y^{-3} = 64$
Work out the value of $\frac{x}{y}$

$\frac{x}{y} =$ ....
(4 marks)

Question 6

Simplify:

$$\frac{b\sqrt{b}}{\sqrt[8]{b}}$$

Leaving you answer in the form:

$$b^n$$

Where n is a fraction to be found.

$n =$ ....

Question 7

Solve $9^{x+4} = 3^{\frac{10}{x}}$.

....

Question 8

Simplify:

$$\frac{(3x^2y^3)^3 \times (2x^4y^{-2})^4}{8\sqrt[4]{16x^{12}y^8}}$$

....

Exercise B:

Find the value of:

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(\frac{1}{9}\right)^{1/2}$

4) 5^{-2}

5) 18^{0}

6) 7^{-1}

7) $27^{2/3}$

8) $\left(\frac{2}{3}\right)^{-2}$

9) $8^{-2/3}$

10) $(0.04)^{1/2}$

11) $\left(\frac{8}{27}\right)^{2/3}$

12) $\left(\frac{1}{16}\right)^{-3/2}$

Simplify each of the following:

13) $2a^{1/2} \times 3a^{5/2}$

14) $x^{3} \times x^{-2}$

15) $\left(x^{2}y^{4}\right)^{1/2}$